Fast Equivalence Relations in Datalog

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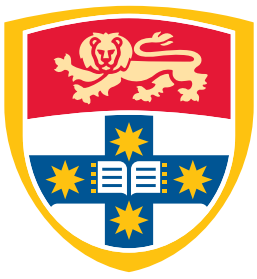

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Abstract

Declarative programs have encountered a resurgence in popularity for their ease of expression for problems that involve large data sets; including static program analysis, security verification, network routing, and more. Of interest to efficiency, are specialised data-structures designed to increase performance within these declarative program evaluation frameworks.

In this thesis we provide and demonstrate an efficient method for representing equivalence relations within Datalog engines, providing at best a quadratic speed-up and space improvements. We do so via the use of modified parallel union-find data-structures and extending the semi-naïve evaluation approach to support these equivalence relations within the solving framework. To our understanding, this is the first time that a self-computing data-structure is deployed in a Datalog engine, i.e., a set of rules is executed implicitly by the data-structure.

We demonstrate the efficacy of this approach by implementing this in the SOUFFLÉ Datalog engine, and comparing to the explicit representation of such programs in Datalog on real-world data-sets. We show that our data-structure is able to store over a trillion tuples for a static program analysis scenario - deriving the final result in under 4 seconds, where we believe an explicit representation of these equivalence relations would take several *years* to compute.

Acknowledgements

I'd like to thank my supervisor, Bernhard, for all the support, knowledge, ~~dashing good looks~~, patience, and wisdom afforded to this project. How he has coped with me over these years is beyond me. I extend my thanks to the PLANG research group; Abdul, David, John, Lexi, Lyndon, Martin (as well as the transient few) - for their meritorious company, presentations, and sanity checks afforded to me, especially that of the past few months.

However, I'd like to reserve my greatest regards to the structural integrity of this building, and for the support it has given to me thus far.

CONTENTS

List of Figures

List of Tables

CHAPTER 1

Introduction

In recent years Datalog has emerged as domain-specific logic programming language for a wide range of applications including static program analysis (Bravenboer and Smaragdakis, 2009), program security (Marczak et al., 2010), program optimisations (Liu et al., 2012), cloud computing (Alvaro et al., 2010), and networking (Loo et al., 2005). Accompanying this resurgence is a wide array of specialised frameworks on Datalog systems that provide specialisations for various problem domains. These problems may be solved through several Datalog-based evaluation engines: LogicBlox, (Aref et al., 2015) SOUFFLÉ , (Scholz et al., 2016) Z3, (De Moura and Bjørner, 2008) and bddbddb, (Whaley and Lam, 2004) - to name a few.

As a declarative programming language, Datalog programs are solely represented by *what* tasks they perform, rather than the low level details of *how*, resulting in reduced complexities in program development and less upkeep. However, as with high-level interfaces, the programs must be sufficiently optimised so that they are competitive with imperative implementations. These modern evaluation engines have approached this in various ways.

Programs may be optimised at the source level, with program rewriting transformations such as Magic Set (Ullman, 1989) that prune unnecessary computation in a Datalog program. In addition, Datalog programs may be parallelised in that queries can be sped up via distributing work across a set of threads or CPUs (Scholz et al., 2016; Lam et al., 2013). However, substantial performance gains can be achieved by specialising the underlying data-structures for storing logical relations in a Datalog engine.

There has been previous successes in representing analyses using specialised data-structures in Datalog engines. These typically take the form of a specialising Datalog engines for a given purpose. Binary Decision Diagrams (BDDs) had been used by Whaley et al. in the `bddbddb` Datalog engine (Whaley, 2004) to produce the first scalable, context-sensitive, inclusion-based, (Andersen, 1994) pointer analysis for Java programs (Whaley and Lam, 2004). Typically, these context-sensitive analyses often include call

graphs with 10^{14} paths or more, (Whaley and Lam, 2004) and storing these explicitly would be intractable even for smaller programs upon which are being analysed. BDDs are an efficient representation of very large logic relations - potentially exponential in size, as is the case in context-sensitive analyses - and enables efficient set operations between these relations. BDDs are an efficient data-structure to compress relations in form of truth-tables. The authors noted that the performance of such generated BDD programs were faster than the manually optimised counterparts. These BDDs are not necessarily all-encompassing - the bddbddb tool relies on a good variable order which cannot be found in polynomial time.

LogicBlox (Aref et al., 2015) is a generalised declarative language system based on Datalog aimed at developing enterprise software, which uses high-performance, trie data-structures to support relations efficiently. The primary motivation for the use of tries is for computing joins between relations via their custom *Leapfrog Triejoin* algorithm. This data-structure is general, in that it is not designed specifically for a certain problem but instead provides fast performance for all interactions with the LogicBlox system. Figure 1.1 demonstrates the space efficiency of storing tuples with many shared prefixes. This may not be the case generally that most relations stored share the same few elements at the front of the tuple. Performing a join in these cases is simple - iterating through all tuples with a prefix involves traversing over the direct children. However, for cases which joins must be performed on the last element in a tuple, this data-structure is less performant.

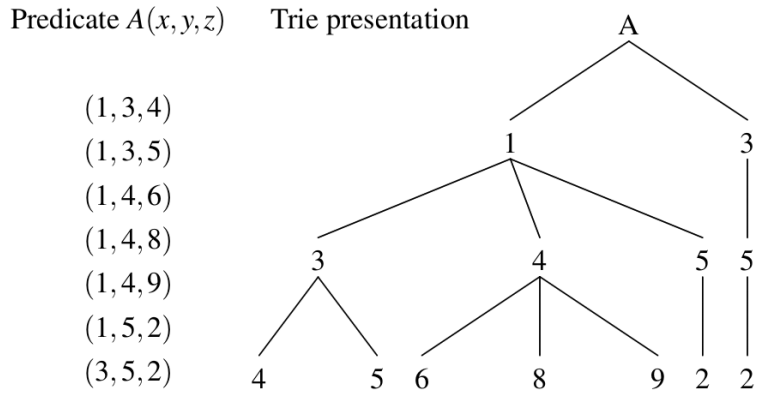


FIGURE 1.1: Trie representation of a ternary relation in LogicBlox (Aref et al., 2015)

LogicBlox is a popular framework for performing large-scale program analysis, performing analyses on specifications that are not feasible for bddbddb (Bravenboer and Smaragdakis, 2009). One shortcoming is that LogicBlox has for a single logical relation a single trie. Manual optimisations are required to

replicate logical relations to achieve high-performance. In addition, LogicBlox evaluates these programs in a single-threaded manner, not exploiting the ever increasing parallelism workloads of modern CPUs. It is a proprietary system and hence not open-source.

The μZ engine is a Datalog engine in the Z3 toolbox (Hoder et al., 2011) that has been developed by Microsoft Research. The μZ datalog engine uses a hashmap to store logical relations (Scholz et al., 2016). Hashmaps adapt poorly to large-scale problems since their randomized memory access destroy cache locality. However, μZ compensates the lack of fine-tuned data-structure by sophisticated source-code level optimisations.

The design and implementation of SOUFFLÉ started at Oracle Labs in Brisbane. It was initially developed to detect security flaws in OpenJDK. Since it was made open-source in early 2016, SOUFFLÉ has been deployed in various applications including the analysis of networks at Amazon, and Ethereum smart contracts. SOUFFLÉ applies code specialization ideas to generate efficient code from Datalog programs. It builds a hierarchy of Futamura projections (Futamura, 1999) to translate Datalog program to an efficient parallel C++ program. The data-structures for the logical relations in a program are specialized depending on the operations that are performed. This approach diverges from prior-art: other Datalog engines use a single data-structure for representing logical relations. As a result, SOUFFLÉ exhibits performance characteristics that are on-par with hand-crafted state-of-the-art tools.

There is a large class of relations in Datalog programs that is a very specific case called equivalence relations. These are binary relations that are reflexive, symmetric, and transitive. Equivalence relations occur in various applications including points-to, network analysis, and smart-contracts to group objects. Current Datalog engines do not use any specialised data-structures to represent equivalence relations efficiently. A union-find data-structure (Kleinberg and Tardos, 2005) would be an appropriate choice to represent logical equivalence relations. Instead of executing horn clauses representing properties of equivalence relations: (1) reflexivity, (2) symmetry, and (3) transitivity; the underlying data-structure executes these clauses implicitly. Hence, no rules need to be executed for properties (1)-(3) and the best case compression reduction using union-find is square-root of the domain size. To achieve the integration of a union-find data-structure in a Datalog engine, several problems arise:

(1) The semi-naïve evaluation strategy for computing the result of Datalog programs is not designed to cope with self-computing sets. The semi-naive evaluation strategy has to be extended so that we self-computing data-structures such as union-find can be used.

(2) The union-find data-structure has the problem that it assumes a dense domain. This is an obstacle for practical use because the domain of an equivalence relation are arbitrary numbers. A densification of the domain is required to deploy efficient union-find data-structure implementations.
(3) Another challenge is the parallelisation of the union-find data-structure and any auxiliary data-structures for storing logical equivalence relations. This is an essential need to harvest the computational power of modern machines.
(4) The union-find data-structure needs to simulate the operations of a binary relations. The implementation of a generic relation interface is required to implement these operations.

The above requirements are very challenging in terms of data-structure/algorithmic design. To show that this is possible, we have implemented specialised equivalence relations in SOUFFLÉ . The objective was to show that specialised data-structures for specific purposes are possible and bring substantial performance to real-world applications.

1.0.1 Contribution

The contributions of this work are as follows:

(1) We extend the semi-naïve evaluation strategy in Datalog to accommodate for self-computing data-structures such as union-find that perform rules implicitly.
(2) We introduce a multi-layered data-structure designed to efficiently represent equivalence relations within Datalog engines.
(3) We develop various parallel multi-threaded data-structures designed for equivalence relations.
(4) We perform extensive experiments to show the efficacy of our approach.

1.0.2 Outline

To begin, we explore relevant material to the topic of this thesis. In Chapter 2 we introduce and formalise Datalog, investigating the evaluation strategies within both top-down and bottom Datalog evaluation strategies and the optimisations that may be made - also covering the SOUFFLÉ environment and equivalence relations.

In Chapter 3, we investigate the layered data-structure, exploring the ADT interfaces, implementation details, as well performing micro-benchmarks for each layer. Our three layers focus on iteration over implicit pairs, transformation of a sparse domain onto a ordinal domain, and the partitioning of equivalence classes through the union-find data-structure.

In Chapter 4 we apply real-world benchmarks to our data-structure and compare them to other approaches, mentioning future work in Chapter 5, and concluding in Chapter 6.

CHAPTER 2

Background

2.1 Datalog

Datalog uses a fragment of first-order predicate logic(Abiteboul et al., 1995; Green et al., 2013b; Greco and Molinaro, 2015). It restricts the model to a finite universe and rules must be phrased as Horn Clauses, i.e., disjunctions of negated atoms with at most one positive atom. As a declarative query language, Datalog provides a method to describe a program with regards to goals, rules, and facts. The Datalog language lets a programmer ignore implementation details, and to focus instead on specifying *what*, not *how*, a program evaluates.

2.1.0.1 Structure

In Datalog we differentiate between two types of logic relations. The first type of relations is called an Extensional Database (EDB) which resembles the input of a Datalog program, i.e., the relations consists of known facts only and have no rules. The second type of relations is called an Intensional Database (IDB). Relations in the IDB are computed relations, i.e., their result is derived from a set of rules. The syntax of a Datalog rule is given below:

$$R_0 \leftarrow R_1, R_2, \ldots, R_n.$$

Each R_j corresponds to a relation, which takes the form $r(x_0, x_1, \ldots, x_n)$ for a relation with an arity of n. Each x_i is either a constant or a variable - note that the set of constants are from a finite universe. We refer to the left-hand side of the rule as the head of the rule and we refer to the right-hand side of the rule as the body. The body of the rule is a series of conjunctions of *atoms*, or simple predicates of existence that imply the head. A rule can be interpreted as a conditional, i.e., if the body $R_1, \ldots, R_n$ of a rule holds, then then head of the rule must hold.

Rules without bodies are called facts, meaning that the head is unconditionally true - these are said to be within the EDB. These are either denoted as '$R_0 \leftarrow .$', or for brevity '$R_0.$'. The atoms R_i are of the form $r_0(X_1, X_2, \ldots, X_n)$, this captures variables $X_1, \ldots, X_n$ that may be shared across atoms. The following Datalog rule $r(X)$ (for a variable X in the finite universe U) is true if $s(X) \wedge t(X)$.

$$r(X) \leftarrow s(X), t(X).$$

So, if $s(\text{“}string\text{”})$ and $t(\text{“}string\text{”})$ holds, $r(\text{“}string\text{”})$ will consequently be true. The atom $s(x)$ may be true for some variable $x \in U$ either through the existence of a fact $s(x).$ within the EDB, or a rule that describes $s(x)$ as true as part of the IDB. We may define a *goal* as a rule that has no head, i.e. $\leftarrow R_1, \ldots, R_n.$. In essence, these are queries, and Datalog programs attempt to evaluate the 'truthiness' of goals.

2.1.0.2 Example Program

The following Datalog program describes a transitive closure over a set of edges (directed as the order of the arguments to each predicate matter) $edge(x, y)$. The rule $path(x, y)$ will generate all reachable y from x through following the set of specified edges. The following program denotes that a path between X and Y exists if there either if there is a direct edge between X and Y, or there is a path from X to Z if there is an edge between X and some other variable Y, and there is a path from Y to Z.

```
1  edge(a,b).
2  edge(a,e).
3  edge(b,c).
4  edge(c,d).
5
6  path(X,Y) ← edge(X,Y).
7  path(X,Z) ← edge(X,Y), path(Y,Z).
```

FIGURE 2.1: Datalog program computing the transitive closure of paths

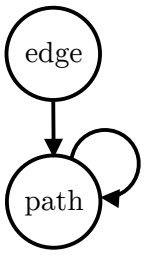


FIGURE 2.2: Precedence graph of Datalog rules

Datalog may contain recursive rules, which allow the above transitive closure over the set of edges to be calculated (and concisely expressed within just two rules). Relational databases alone cannot express

such a rule purely within the relational algebra and calculus upon which they are described. (Aho and Ullman, 1979) Figure 2.2 shows the precedence graph of the above program in Figure 2.1.

Negation in logic is problematic because it can introduce the classical paradox negating itself. For example, the rule $A(x) \leftarrow \neg A(x)$. would define the contents of the relation as a negation of itself. This is logically not sound and cannot be computed. To get a handle on the problem, the research community introduced the notion of stratified negation, i.e., a Datalog program is decomposed in individual strata and ordered according to a fixed order. Negation can only be used if the negated relation is computed in a prior stratum. Stratified negation is the standard method of dealing with negation in Datalog (Abiteboul et al., 1995).

2.1.1 Evaluation

There are two fundamental evaluation strategies for Datalog, i.e., *top-down* and *bottom-up* evaluation. In top-down evaluation, the goal is expanded by rules/facts until the goal is proven, otherwise known as resolution. Top-down methods have been extended for Datalog to make them deterministic and efficient. (Saptawijaya and Pereira, 2013) However, the expansion of the goal relies on symbolic rewriting of rules via resolution. Top-down evaluation works by being provided a relation that must be proven, and evaluating the rules that must hold in order for the head of said relation to hold. For the above example in Snippet 2.1, evaluating the query `?- path(a,c)` must be done firstly through an invocation of the second path rule (Line 7) (the first rule (Line 6) does not hold as there is no direct `edge(a,c)` statement). There exists a single variable assignment that holds for the first atom in the body, namely `edge(a,b)`. All that is left to prove consequently is `path(b,c)`, which trivially holds from the first rule, as it exists in the EDB for `edge`. No irrelevant atoms were be evaluated in order to test the query. Negation, that is requiring a tuple does not exist, will require all knowledge for that rule to be computed to ensure monotonicity of new knowledge. If large number of tuples are either generated in the IDB and/or stored in the EDB, the top-down approach computationally collapses and becomes intractable for large-scale problems (Green et al., 2013a).

To overcome the limitations of top-down evaluation strategies, modern Datalog engines (Aref et al., 2015; De Moura and Bjørner, 2008; Whaley, 2004) use bottom-up evaluation strategies in conjunction with magic-set techniques (Abiteboul et al., 1995; Ullman, 1989). The bottom-up evaluation strategy exploits the connection to lattice theory and logic. Logical computations can be expressed over a sub-set lattice and a consequence operator, which is a monotonic function. The fixed-point of the immediate

consequence operator coincides with the result of the Datalog program; Knaster–Tarski's (Tarski, 1955) theorem is fundamental to make this connection work.

The immediate consequence operator $\mathcal{T}_{P_D}$, for a program P_D is defined as follows:(Greco and Molinaro, 2015)

$$\mathcal{T}_{P_D}(I) = \{A_0 | A_0 \leftarrow A_1, \ldots, A_n \text{is a ground rule in } ground(P_D) \text{ and } A_i \in I \text{ for every } 1 \leq i \leq n\}$$

This is the set of ground atoms that are generated as an immediate consequence of I w.r.t. P_D, iterating this until a fixed-point will result in the set of all computable tuples. Bottom-up evaluation involves evaluating all captured rules in order to reach the target rule. We are able to perform some optimisations, for example we don't consider rules that do not reach the goal vertex in the precedence graph. The following example demonstrates an approach of bottom-up evaluation wherein new knowledge is learned for each iteration, although also that this approach is inefficient as tuples that already exist are re-evaluated per step. We use the same example program shown in Figure 2.1, with $path_1$ referring to the first rule for $path$, and $path_2$ referring to the second.

2.1.1.1 Naïve Evaluation

Using the immediate consequence operator $\mathcal{T}$, we demonstrate a naïve method of solving Datalog bottom-up. Initially, I_0 contains all the ground atoms that comprise the EDB of P_D as they are the immediate consequence of no deduced knowledge, and thus can be used in the first iteration to generate new knowledge. All previous knowledge is used to deduce further knowledge. We use the $\bowtie$ operator to denote a join across relations - i.e. this will match relations with matching terms in their arguments, `edge(a,b), path(a,c)` match on the first argument, for example.

$$I_0 = \mathcal{T}_{P_D}(\emptyset) = \{edge(a,b), edge(a,e), edge(b,c), edge(c,d)\}$$

$$I_1 = \mathcal{T}_{P_D}(I_0) = I_0 \bowtie path_1 \cup I_0 \bowtie path_2 = I_0 \cup \{path(a,b), path(a,e), path(b,c), path(c,d)\}$$

$$I_2 = \mathcal{T}_{P_D}(I_1) = I_1 \bowtie path_1 \cup I_1 \bowtie path_2 = I_1 \cup \{path(a,c), path(b,d)\}$$

$$I_3 = \mathcal{T}_{P_D}(I_2) = I_2 \bowtie path_1 \cup I_2 \bowtie path_2 = I_2 \cup \{path(a,d)\}$$

$$I_4 = \mathcal{T}_{P_D}(I_3) = I_3 \bowtie path_1 \cup I_3 \bowtie path_2 = I_3$$

As $I_3 = I_4$, we have reached a fixed point, and thus there is no new knowledge to be gained. Redundant computation is performed at each application, where *all* previous knowledge will be regenerated based on the current knowledge each iteration.

Bottom-up strategies have the disadvantage that the totality of all IDB tuples are to be computed. To reduce the amount of data in the IDB, the magic-set transformation (Bancilhon et al., 1985; Abiteboul et al., 1995) was introduced. The transformation rewrites the Datalog program according to its goal so that unnecessary tuples in the IDB will not be computed. We explore a more efficient method of bottom-up evaluation, *semi-naïve evaluation*, in the next section.

2.2 Semi-naïve Evaluation

Naive evaluation (as demonstrated with the consequence operator) does not consider the dependencies of the recursive relations in a Datalog program, hence, the evaluation is performed globally over all rules in the program. With a large number of relations which may be only partially mutual recursive to each other, the naive evaluation strategy becomes expensive. Hence, the evaluation is broken up into an evaluation over different strata, i.e., each strata is computed in an own fixed-point, and the order of the strata is given by the precedence graph. The other extension of the semi-naive evaluation strategy is that new tuples in the previous iteration of the fixed-point calculations are memoized. With that "new knowledge" less computations can be performed for gaining the new knowledge of the current iteration.

Semi-Naive evaluation was very successful in various state-of-the-art Datalog engines including LogicBlox (Aref et al., 2015), μZ/Z3 (Hoder et al., 2011; De Moura and Bjørner, 2008), bddbddb (Whaley, 2004), and SOUFFLÉ (Jordan et al., 2016). SOUFFLÉ is a high performance Datalog interpreter & compiler, converting a superset of the Datalog language to high performance, parallel C++ code. In order to compute rules, SOUFFLÉ performs bottom-up evaluation. Rather than computing all the relations naïvely, SOUFFLÉ uses this semi-naïve evaluation, which when sufficient program optimisations are made to a Datalog program, will outperform or equal any top-down evaluation strategy for *any* query over a program. (Ullman, 1989) Semi-naïve bottom-up evaluation is also conducive to parallelism - it is not difficult to concurrently search for rules and perform joins across the knowledge.

2.2.1 Rule Transformation

Semi-naïve evaluation involves transforming each rule into a set of new rules. Consider the rule:

$$R_0 \leftarrow R_1, \ldots, R_n.$$

For each R_i we may create new relations for each iteration k: R_i^k, and ΔR_i^k, and $newR_i^k$. ΔR_i^k is an incremental rule, it contains instances that were only exclusively newly created in iteration k, R_i^k denotes the tuples of R_i known in iteration k, while $newR_i^k$ represents tuples that were generated in iteration k.

The new rules are constructed such that tuples will only be generated if they depend on a tuple generated exclusively in the previous iteration, so we generate a series of rules from the original rule such that they all involve a delta rule of the body atoms. We represent the relations using relational algebra.

For each iteration, $k+1$ we solve over the following:

$$newR_0^{k+1} = (\Delta R_1^k \bowtie R_2^k \bowtie \ldots \bowtie R_n^k) \cup (R_1^k \bowtie \Delta R_2^k \bowtie \ldots \bowtie R_n^k) \cup (R_1^k \bowtie \ldots \bowtie R_{n-1}^k \bowtie \Delta R_n^k) \backslash R_n^k$$

As a result, $newR_0^{k+1}$ contains newly derived tuples from iteration $k+1$, after we have processed all the rules for iteration k, we may update our delta rule to consist of the new knowledge derived this iteration: $\Delta R_0^{k+1} = newR_0^{k+1}$. We update the total known knowledge as $R_0^{k+1} = R_0^k \cup newR_0^{k+1}$ each iteration.

For relations with no rules in the IDB (i.e. they are only specified within the EDB), it is not necessary to create delta rules - their set of tuples will not change which results in an empty delta each iteration, and it is redundant to join over the empty set.

We do not simply iterate over all rules within the program within a single iteration. Semi-naïve follows the precedence graph, in that rules should be computed after their ancestors. For recursive rules (including mutually recursive rules), this is slightly different.

If a rule is not part of a cycle in the precedence graph (self-recursive or mutually recursive), there is no need to compute delta knowledge, as the rule will reach a fixed point after a single iteration. For rules that are part of a cycle in the precedence graph, only relations that are part of the SCC require delta knowledge.

For the given Datalog program (EDB omitted) in Snippet 2.1, we demonstrate the precedence graph in Figure 2.3, and the necessary semi-naïve rules.

```
b(x) :- a(x).
b(x) :- c(x,x).
c(x,y) :- d(x,y), c(x,y).
d(x,y) :- c(x,y).
```

LISTING 2.1: Example Recursive Datalog Program

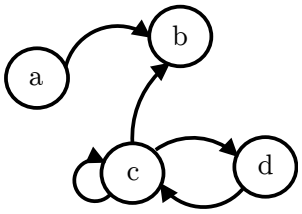


FIGURE 2.3: Precedence graph for the rules within the Datalog program in Snippet 2.1

An example ordering may be $a \rightarrow \{c, d\} \rightarrow, b$. Note that c and d are solved at the same time, i.e. an iteration will consist of deriving tuples for both c and d - this is due to their mutual recursion - generation of facts for one may consequently generate facts for the other. We do not require special constructed rules for a nor b. Each iteration $k + 1$ for solving c, d consists of iterating over the following, until a fixed-point is reached.

$$newc^{k+1} = (\Delta d^k \bowtie c^k) \cup (d^k \bowtie \Delta c^k) \setminus c^k$$
$$newd^{k+1} = \Delta c^k \setminus d^k$$

2.3 Equivalence Relations in Datalog

Equivalence relations are binary relations that are reflexive, symmetric, and transitive. Any elements that are related by virtue of these properties are to be considered within the same equivalence class. As Datalog allows concise expression of relations, it is clear that expressing equivalence relations is also simple. We include a binary relation with equivalence relation semantics in Figure 2.4.

```
relation(a,a) :- relation(a,_).          // reflexive
relation(a,b) :- relation(b,a).          // symmetric
relation(a,c) :- relation(a,b), relation(b,c). // transitive
```

FIGURE 2.4: Equivalence relations may be defined via adding three rules in Datalog

From a single tuple added, many tuples may be consequently derived. If as part of this program's EDB, there were the rules: `relation(1,2)`, the consequent knowledge of `relation` would be:

`relation(1,1),relation(1,2),relation(2,1),relation(2,2)`

If the EDB also now contained `relation(2,3)`, 5 additional tuples would be part of the final computed knowledge:

`relation(1,3),relation(2,3),relation(3,1),relation(3,2),relation(3,3)`

This 'worst-case' behaviour is exhibited when a single large equivalence class is part of the computed facts. In the above example, only a single equivalence class exists - $E = \{1, 2, 3\}$. If we also added in `relation(4,5)` into the EDB, we would consequentially have an addition 4 facts within the computed knowledge - the equivalence classes in the program would now be $E = \{\{1, 2, 3\}, \{4, 5\}\}$. The number of pairs in the equivalence relation is the sum of the square of the sizes of equivalence classes within the relation:

$$\sum_{i \in E} |i|^2$$

There are multitudes of use-case scenarios for equivalence relations in Datalog, we cover two in our real-world benchmarks (Section 4). In addition to computing Bitcoin user-groups, and Steensgaard points-to analyses, equivalence relations can be used to compute SCCs in graphs (Suthers, 2015); must-alias pointer analyses (Kastrinis et al., 2018); computing optimal networking routes (Thau Loo, 2010) and more.

Snippet 2.2 demonstrates an example Datalog program that contains an equivalence relation. The EDB consists of three facts on lines 1-3. Figure 2.5 shows the EDB facts in black edges in the graph, whilst the derived facts are shown in red. The program defines rules and facts for whether two people live in the same suburb.

```
same_suburb(alice,bob).
same_suburb(charlie,bob).
same_suburb(derek, eve).

same_suburb(X,Y) :-
    same_suburb(X,_).
same_suburb(X,Y) :-
    same_suburb(Y,X).
same_suburb(X,Z) :-
    same_suburb(X,Y),
                same_suburb(Y,Z).
```

LISTING 2.2: Same suburb equivalence relation in Datalog

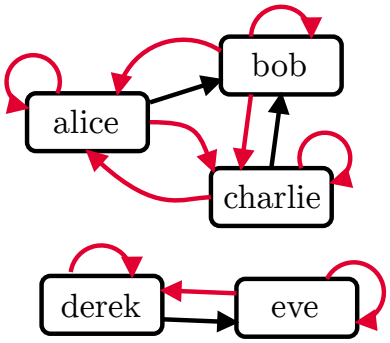


FIGURE 2.5: Explicit facts are denoted in black (EDB), derived facts are in red

There are many derived facts in this example - from the declaration of 3 facts within the EDB, 10 facts are evaluated as a consequence as a result from solving over the IDB (pictured in Figure 2.5 as red edges).

Any recursive evaluation strategy may require the traversing/evaluation of long paths, can result in a number of solving iterations linear to that path length. The transitive rule of any specified equivalence relation will incur this potential penalty.

2.4 SOUFFLÉ Syntax

SOUFFLÉ is a new Datalog engine that was designed for large-scale program analysis. The language of SOUFFLÉ introduces a superset of Datalog containing types, functors including built-in arithmetic predicates (such as infix addition: `x + 1`), aggregations (sum, average, string concatenation), records, and more. With functors the evaluation of SOUFFLÉ programs becomes Turing complete (Keynes, 2017), i.e., non-terminating logic programs can be constructed - this enables SOUFFLÉ to be fully-expressive. A simple example of such a program is demonstrated in Snippet 2.3 which calculates the successor of a number ad-infinitum.

LISTING 2.3: Non-terminating Datalog program

```
.decl succ(x : number)

succ(0).
succ(x+1) :- succ(x).
```

```
5 .output succ
```

In the above snippet, several important syntactical features are demonstrated:

- **Types:** each argument of user defined predicates uses types, which enforces type-correctness between predicates. In this example, `x` is a numeric type
- **Predicate Declarations:** each rule defined must be declared with arity and types (line 1)
- **I/O Qualifier:** each predicate may be marked to output all rules to file (`.output`, line 5) or to read facts in from a file (`.input`)

Output rules imply that all tuples for that rule must be generated - we do not specify goals in SOUFFLÉ , only that a relation must be fully computed. Goals can be emulated via creating a relation to be output whose body consists of the goal to be proved. New types can also defined via the `.type XXX` statement. Other functionality and syntax features are not covered in this article - the above is enough to describe all future snippets of Datalog.

CHAPTER 3

Equivalence Relations in Datalog Engine

The major motivation of this work is to use specialised equivalence data-structure in a parallelised Datalog engine. The specialised equivalence data-structure performs the rules for reflexivity, symmetry, and transitivity in-situ, i.e. the rules do not need to be performed as part of the semi-naïve evaluation explicitly. The advantage of this approach is that substantial performance gains are to be expected. For this purpose, the Datalog engine needs to be modified so that the specialized equivalence data-structures can be used seamlessly. There are two major undertakings in the design and implementation of equivalences. The first undertaking is the modification of the semi-naïve evaluation approach, i.e., how can specialised equivalence relations be used in conjunction with the semi-naïve evaluation approach. The second undertaking is the design and implementation of a parallel equivalence data-structure for equivalence relations that behaves like any other logical relations.

The formal definition of an equivalence relation is as follows: For elements within the domain D, an equivalence relation may hold binary tuples $R = (a, b) \in D \times D$, where the existence of tuples are subject to the elements a and b existing within the same equivalence class. The equivalence classes is defined as a binary relation, that is,

$$\forall a \in D, (a, a) \in R$$

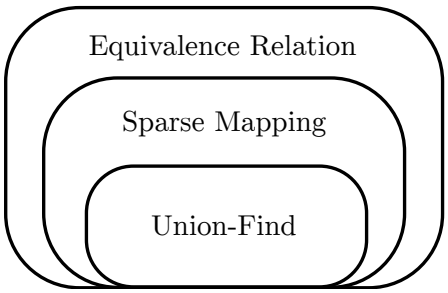
$$(a, b) \in R \Rightarrow (b, a) \in R$$

$$\text{for } a, b, c \in D : (a, b), (b, c) \in R \Rightarrow (a, c) \in R$$

The problem with the definition is that the domain of the binary relation could be quite large. For example, if the binary relation is defined over the set of natural numbers. However, only a small subset of numbers may be actually used in the concrete instance of a relation. Hence, we introduce a condensation of the domain, i.e., we map the Datalog domain to an ordinal domain. The ordinal domain of a domain numbers the elements of the original domain starting with zero. Hence, the ordinal domain of a domain densifies

the original values of the equivalence relation into dense ranges from zero to the number of elements used in a concrete instance of a binary relation.

To efficiently express equivalence relations in Datalog, we propose a three layered data-structure for equivalence relations. Each layer serves a purpose: The equivalence relation layer allows iteration over the implicit pairs, insertion, and extension of tuples; the sparse mapping layer allows the storage of sparse values efficiently; and the disjoint set layer provides a wait-free Union-Find data-structure, to store equivalences between elements.



- **Union-Find:** Store elements of same relation in a same-set structure
- **Sparse Mapping:** Provide value abstraction
- **Equivalence Relation:** Allow iteration over all equivalence tuples

FIGURE 3.1: Architecture

Pictorially - elements within the same equivalence relation are stored densely encoded, as seen in Figure 3.4, on top of which a sparse-representation of the disjoint can be formed (Figure 3.3) that requires the storage of the following bijective mapping:

$$e \Leftrightarrow 1, f \Leftrightarrow 5, b \Leftrightarrow 4, a \Leftrightarrow 3, c \Leftrightarrow 2, d \Leftrightarrow 0$$

Figure 3.2 shows the corresponding pairs of the stored equivalence relation, for each equivalence class the number of tuples is the square of the size of the equivalence class.

(a, a)

$(b, b), (b, e), (b, f), (e, b), (e, e)$
$(e, f), (f, b), (f, e), (f, f)$

$(c, c), (c, d), (d, c), (d, d)$

FIGURE 3.2: Pairs of the equivalence relation

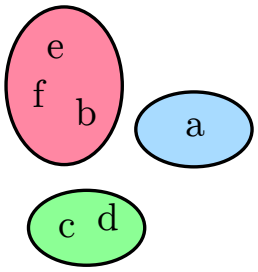


FIGURE 3.3: Disjoint sets of sparse elements

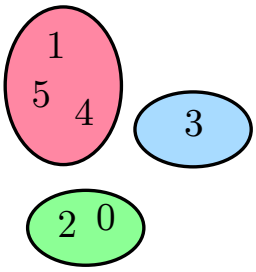


FIGURE 3.4: Disjoint sets of elements, tightly encoded

3.1 Equivalence Relation Layer

An equivalence relation provides an Abstract Data-Type (ADT) to expose all pairs in the equivalence relation and to insert new pairs to it. The interface is designed such that it mimics the functionality of a binary relation that is explicitly stored. The operations of the ADT are very basic. The data-structure is composed of various layers to perform the necessary operations of the equivalence relation. The data-structure layers contains a generic interface mimicking a generic logical relations, a Densifier (discussed later in more detail in Section 3.2) that compresses the domain of the equivalence relation to an ordinal domain, and a parallel union-find data-structure to partition the elements into disjoint-sets.

To accommodate semi-naïve evaluation for equivalence relations, we must extend the behaviour for the delta relations. Delta relations contain the new knowledge learned in the previous iteration, done so in order to reduce the number of redundant calculations, as the generation of new knowledge only occurs when considering the most recently learned facts. However, if this delta knowledge was stored explicitly (i.e. all pairs are stored), we would lose out on some benefits of using the implicit representation for the underlying relation. Thus we stored this delta knowledge also as an equivalence relation.

We extend the definition of the delta operation to be an over-approximation, that is, the delta knowledge may include pre-computed knowledge from prior iterations. This can reduce the efficiency of the semi-naïve evaluation, and for the worst-case, revert to the performance of naïve evaluation. By treating the delta relation as an equivalence relation, this allows efficient generation of the updated relation (i.e. R^{k+1}).

Our new definition for our equivalence relation delta - Δ_{eqrel} is:

$$\Delta_{eqrel} R_i^{k+1} = newR_i^{k+1} \odot R_i^k$$

The $\odot$ operator is the extension between the two equivalence classes such that the equivalence classes are merged. A pictograph of the resulting delta relation is demonstrated in Figure 3.5. Superfluous edges are marked in blue, these comprise the over-approximation. Note that reflexive relations, i.e. self-loop edges are omitted from the graph for brevity.

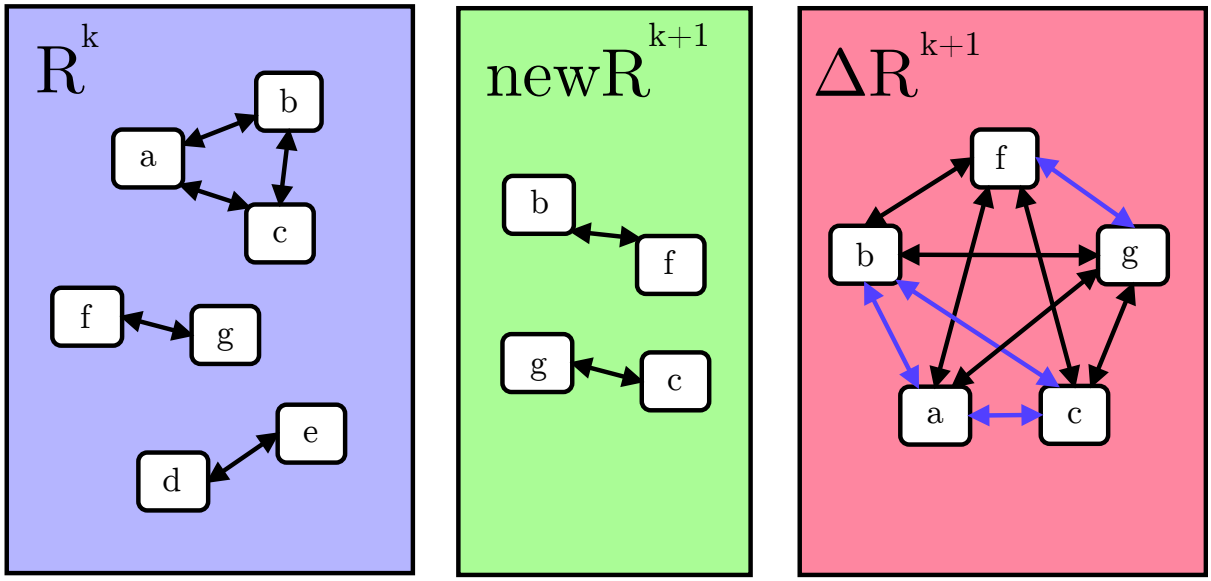


FIGURE 3.5: Resulting delta relation after the extension

These edges in the delta relation are not explicitly added during the extension step. Instead, we perform the algorithm in 1 to extend a relation with another, also ensuring that no irrelevant classes are kept. The equivalence class $\{d, e\}$ is thus excluded from the resulting delta relation based on the example given in Figure 3.5.

We supply the arguments R^k and $newR^{k+1}$ as *origR* and *newR* respectively. Firstly, we iterate over the new knowledge, $newR^k$, and create elements representing each element that occur in *both* R^k and R^{k+1}. We store these in a temporary set to ensure that we don't cover existing elements multiple times.

For each element in this set, we add their corresponding equivalence classes into the new delta relation for both old and new relations. We simply find the representative in that set, and insert a tuple between that representative and the other elements within the set. This algorithm operates in amortised $\mathcal{O}(\alpha n)$

time - each element is visited at most once in each relation, where it will at most perform a constant number of `find` or `union` queries to find the representatives of a class, or to add an edge (i.e. insert a pair into a relation).

Algorithm 1 Return an extended relation

```
1:  procedure EXTEND(origR, newR)
2:      new-relation ← empty relation
3:      element-list ← empty set
4:      ▷ Add elements that exist in both sets to our worklist
5:      for element ∈ newR do
6:          if element ∈ origR then
7:              add element to element-list
8:          end if
9:      end for
10:     ▷ add all classes from oldR that contain an element from element-list
11:     for element ∈ element-list do
12:         class ← equivalence class that contains element
13:         for child ∈ class do
14:             insert (element, child) into new-relation
15:             ▷ Ensure we don't visit a class twice
16:             if child ∈ element-list then
17:                 remove child from element-list
18:             end if
19:         end for
20:     end for
21:     ▷ add all classes within newR
22:     for class ∈ newR do
23:         representative ← representative of class
24:         for element ∈ class do
25:             insert (representative, element) into new-relation
26:         end for
27:     end for
28:     return new-relation
29: end procedure
```

The read operations of the data-structure deals with accessing the pairs of the equivalence relations. The access has different modes depending whether the first, the second, or both elements of the pair are fixed. The read access itself is performed via an iterator. The iterator has the usual operations of checking whether all tuples have been iterated (`HASNEXT()`), advancing to the next tuple (`NEXT()`), and accessing the current tuple (`GETELEMENT()`).

Note that for the semi-naïve evaluation strategy, we do not need to support for simultaneous read and write operations. The semi-naïve evaluation strategy has two distinct phases, which are entered via

barriers. These phases are alternating. The first phase may have multiple concurrent reads but no writes. The second phase has multiple concurrent writes but no reads. This observation alleviates the design of the data-structure and simplifies issues which would normal occur in concurrent data-structures.

3.1.1 ADT

Provided this layer is an abstraction layer, we must forward on appropriate function calls to the lower layers, modifying them as required. We also provision an iterator interface, that is, providing methods to call that will allow iteration over the tuples in certain fashions, or modes. SOUFFLÉ requires the support of 4 different modes:

- **`ALL()`:** iterate over all pairs in the equivalence relation $(*, *)$
- **`PREFIX(`**α**`)`:** iterate over all pairs where the first element of the pair is fixed $(\alpha, *)$
- **`SUFFIX(`**β**`)`:** iterate over all pairs where the second element of the pair is fixed $(*, \beta)$
- **`FACT(`**α**`,`**β**`)`:** check the existence of a pair fixing both elements with concrete values (α, β)

These iterators are necessary to enable efficient joins within SOUFFLÉ . Joins can be simply performed via finding matching tuples when elements are fixed within tuples.

We also require a `PARTITION(d)` function, that will provide a number of iterators that when iterated over will explore the entire set of tuples implicitly stored within the equivalence relation. The parameter d is a hint, whereby approximately d iterators will be returned, that cover different (in our case, disjoint, but not necessarily the same size) portions of the tuples of the equivalence relation.

In order to modify & query the set of tuples, the following operations are also provided:

- **`INSERT(x,y)`:** insert the tuple (x, y) into the equivalence relation, implicitly inserting tuples implied by the reflexivity, symmetry, and transitivity.
- **`CONTAINS(x,y)`:** return whether the tuple (x, y) exists within the equivalence relation.
- **`INSERTALL(other)`:** insert all tuples contained within the relation *other*. We have specialisations for when `other` is an equivalence relation.
- **`EXTEND(other)`:** perform delta extension for this relation, merging the equivalence relation `other` into this relation, resulting in the minimal delta equivalence relation.
- **`CLEAR()`:** reset this equivalence relation to contain no tuples, emptying it.
- **`SIZE()`:** calculate the number of tuples implicitly stored within this equivalence relation.

We will use the term *anterior* to refer to the first term in a tuple, whereas *posterior* will refer to the latter.

3.1.2 Iteration

The iterators simulate an explicitly stored binary relation. However, the equivalence relation is stored implicitly, and as a result the algorithms for the enumeration of these binary pairs are more involved. We fetch and store the disjoint-sets induced by the union-find data-structure into individual lists, and upon these we iterate over. In Figure 3.6, an example iterator design is shown. We include markers for the current list (coloured blue), and current anterior (green caret) and current posterior (red star) of the present tuple. These lists act as a cache for the iterators to iterate over.

These lists are not necessarily ordered, either internally or externally, except they do require some kind of iteration over them. For each iterator, there are three operations: `NEXT()`, `HASNEXT()`, and `GETELEMENT()`.

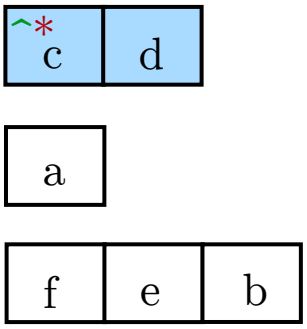


FIGURE 3.6: Equivalence Cache that corresponds to the equivalence classes pictured in Figure 3.7

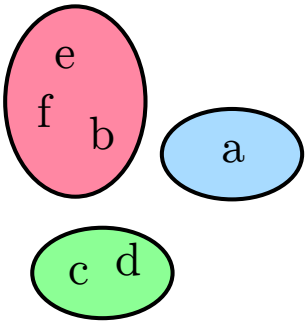


FIGURE 3.7: Example equivalence class partitioning

The approach for iterating over all pairs (`ALL()`) is simple, as seen in Algorithm 2; we advance the *posteriorIterator* (red star *) until we hit the end of the list (indicated in blue). At that point, the *anteriorIterator* (green caret ^) is stepped along one position. This repeats until both the *anteriorIterator* and *posteriorIterator* reach the end of the list, at which point, we move onto the next list and reset the *anteriorIterator* and *posteriorIterator* to point to the start of the now current list. We repeat this procedure until we run out of lists.

Algorithm 2 Advance the iterator - `ALL()`

```
1:  procedure NEXT( )
2:      Advance posteriorIterator
3:      if posteriorIterator is at the end of the current list then
4:          Advance anteriorIterator
5:          if anteriorIterator is at the end of the current list then
6:              Advance to the next list
7:              if there is no next list then
8:                  ▷ We have no more to iterate
9:                  return failure
10:             end if
11:             Move anteriorIterator to the start of the current list
12:         end if
13:         Move posteriorIterator to the start of the current list
14:     end if
15: end procedure
```

The logic for whether there is a next element simply checks whether stepping will cause the iterator to overflow (reach the end of the list).

Algorithm 3 Check whether there is a next tuple - `ALL()`

```
1: procedure HASNEXT( )
2:     if anteriorIterator at end of current list and posteriorIterator at end of current list and there is
   no next list then
3:         return false
4:     else
5:         return true
6:     end if
7: end procedure
```

Retrieving the tuple is retrieving the elements that *anteriorIterator* and *posteriorIterator* point to, and pack them into the front and back of a tuple, respectively.

Algorithm 4 Retrieve the current tuple from the iterator - `ALL()`

```
1: procedure GETELEMENT( )
2:     return (element at anteriorIterator, element at posteriorIterator)
3: end procedure
```

For iteration over tuples in the equivalence relation with fixed anterior (`PREFIX(`α`)`), we fix both the *anteriorIterator* and the current list. When the *posteriorIterator* reaches the end of the current list, this marks the end of the iterator.

Algorithm 5 Advance the iterator - `PREFIX(`α`)`

```
1: procedure NEXT( )
2:     Advance posteriorIterator
3:     if posteriorIterator is at the end of the current list then
4:         ▷ We have no more to iterate
5:         return failure
6:     end if
7: end procedure
```

Whilst the `GETELEMENT()` function remains the same between these two iterator modes, similar changes must be made to the `HASNEXT()` predicate:

Algorithm 6 Check whether there is a next tuple - `prefix(`α`)`

```
1: procedure HASNEXT( )
2:     if posterioriterator at end of current list then
3:         return false
4:     else
5:         return true
6:     end if
7: end procedure
```

For the `SUFFIX(`β`)` iterator, this is nearly identical. Instead, the *posteriorIterator* is fixed to point to the element β, whilst the *anteriorIterator* now advances on successive `NEXT()` invocations. `FACT(`α`,`β`)` is also implemented as an iterator, but both the *posteriorIterator* and *anteriorIterator* are fixed, the current list is also static; `HASNEXT()` will always return false, and attempting to call `NEXT()` will result in failure.

For iterators with fixed terms (as is used internally during joins), it is efficient to find the corresponding list in practice, as the list is stored in the cache hash-map, with representatives of the equivalences classes as keys which can be found in near constant time.

3.1.3 Cache Generation & Implementation

We apply caching as a main technique to accelerate the processing of iterators for the read-phase. Caches are volatile, i.e., if in the next write-phase pairs are added, the caches are invalidated. Figure 3.6 shows an abstract representation of the cache, in practice the lists are stored as values within a hash map, with *representatives* (discussed further in 3.3) of the disjoint-set as the keys. We design the cache such that it

is efficient over all iterator operations, and using thread-safe data-structures such that the construction of this cache can be performed in parallel.

Generating the cache requires only a single pass over the underlying disjoint-set. As the disjoint-set is stored within a union-find data-structure (described further in Section 3.3) as a forest, each tree within that forest contains a single root which is known as the representative for that set. We iterate over all of the elements within the set, and insert that element into a list dictated by the representative of the element's disjoint-set, resulting in elements within the same disjoint-set being inserted into corresponding lists. This can be done in parallel, if representative to list mapping is done via a thread-safe hash-map, provided the list is also thread-safe.

Algorithm 7 details the general process of generating the cache. We directly interact with lower layers; iterating over the dense values stored within the union-find data-structure (named *disjoint-sets* in the following snippet), and also mapping these to their respective sparse-values via the dense to sparse mapping in the Densifier layer (using the *toSparse* function).

Algorithm 7 Generate the equivalence cache

```
1:  procedure GENERATE-CACHE( )
2:      for element ∈ disjoint-set do
3:          representative ← disjoint-set.find(element)
4:          sparse-rep ← toSparse(representative)
5:          sparse-element ← toSparse(element)
6:          if sparse-rep ∉ cache then
7:              cache[sparse-rep] ← empty list
8:          end if
9:          cache[sparse-rep].append(sparse-element)
10:     end for
11: end procedure
```

As the above algorithm is designed to be distributed across parallel workloads, in the actual implementation we iterate over the elements by assigning threads portions of the disjoint set array to independently iterate over. This construction is simple in OpenMP, wherein a preprocessor macro can be added to enable this functionality. The disjoint-sets are stored within a contiguous array, where elements are assigned indexes to represent a value. A shortened version of the parallel C++ code is provided below in Snippet 3.1. We use the Intel TBB `tbb::concurrent_hash_map` (Intel, 2017) in order to provide a thread-safe and fine-grained locking mechanism which simplifies the process for creating the list if it does not exist already. For the list, we use a custom thread-safe random-access list, as described in Section 3.3.3.

LISTING 3.1: C++ parallel cache generation

```
if (isCacheInvalid) {
   const size_t sets = ds.size();
   #pragma omp parallel for   // dictate that this loop can be performed
      in parallel
   for (size_t i = 0; i < sets; ++i) {
      size_t rep = ds.find(i);
      size_t sparseRep = sds.toSparse(rep);
      size_t sparseEl = sds.toSparse(i);

      bool exists = cache.count(sparseRep);
      // thread-safe list creation
      if (!exists) {
         // this acts as a fine grained writer's lock
         accessor a;
         bool exists = cache.insert(a, sparseRep);
         // only create the list if it doesn't exist
         if (!exists) a->second = make_list();

         a->second->append(sparseEl);
      } else {
         // fine-grained reader's lock
         const_accessor a;
         cache.find(a, sparseRep);

         a->second->append(sparseEl);
      }
   }
   cache.rehash();
}
```

We have found through internal profiling, that we observe a sub t-linear run-time improvement for t threads. We suspect this is due to the current method of iterating over the disjoint set list as is default for OpenMP. In this example, OpenMP uses a scheduler to assign the iterations to each thread pool (Lockman,

2013), however, by modifying the loop such that each thread worked on contiguous blocks we believe we would see a reasonable speed-up.

The use of the `isCacheInvalid` variable is to only generate the cache when necessary. The cache is designated to be stale if there has been a modifying operation on the equivalence relation - the following set of operations invalidate the cache (and thus unconditionally setting `isCacheInvalid` to be true): `INSERT`, `INSERTALL`, `EXTEND`, and `CLEAR`. If we wanted to avoid cache invalidation, this would require being able to merge sets of lists quickly, including finding *which* lists to merge, which is the purpose of the union-find data-structure as used in the lowest layer. The `isCacheInvalid` variable must be atomic to prevent contending threads corrupting the stored value.

Due to the behaviour of the implemented hash-map, we must rehash the hash-map prior to iteration. In the case of Intel's TBB, not doing so leads to some values skipped or repeated during iteration. (Nappa, 2018)

For the `size()` operation, this cache generation must be performed prior. In order to calculate the number of pairs, we must know the size of each disjoint-set. The size is simply the sum of the square of sizes for each disjoint set. The other operations `PARTITION`, and iterators (`ANTERIOR, etc`) also require the generation of the equivalence cache.

3.1.4 Partitioning

As the `partition(count)` function is designed to generate a number of iterators over the equivalence relation (so as to allow parallel iteration over a number of threads), we require additional iterator types to divide the number of pairs to iterate over fairly. The `count` argument is only meant as a guide as to how many partitions should be created - this value is by default 400, despite the degree of parallelism usually used by SOUFFLÉ programs being less than 64, due to the cost of sharing data across multiple socket machines. (Scholz et al., 2016).

The new iterator `CLOSURE(d)` creates an iterator per disjoint set. This is beneficial as the iterator now operates solely on a single list, is the only iterator to exist for that disjoint set. The number of pairs covered within a `CLOSURE` iterator is the square of the size of that disjoint set, and can potentially be used to divide work when the number of disjoint sets are large.

The heuristic we use for generating these partitions is simple, as demonstrated in Algorithm 8. Constructing the partitions is greedy - if there are too many disjoint sets there will be a single iterator over each equivalence class (using `CLOSURE`), otherwise, we iterate over each equivalence class (disjoint-set), and if it is large we split up the disjoint-set into many iterators, one for each element in the class (using `ANTERIOR`). If the class is small, we generate a `CLOSURE` iterator for that disjoint-set. We currently do not have the ability to group multiple equivalence classes within a single iterator.

Algorithm 8 Partition the equivalence relation to a number of iterators

```
1:  procedure PARTITION(numIterators)
2:      iterators ← empty list
3:      ▷ Supply an iterator per equivalence class
4:      if number of equivalence classes ≥ numIterators then
5:          ▷ Add a closure iterator for the representative of each set
6:          for class ∈ equivalence classes do
7:              rep ← REPRESENTATIVE(class)
8:              iterators.append(CLOSURE(rep))
9:          end for
10:         return iterators
11:     end if
12:     totalPairs ← number of pairs within the equivalence relation
13:     for class ∈ equivalence classes do
14:         ▷ if this class needs to be split up into smaller ones
15:         if class.size ≥ totalPairs / numIterators then
16:             for element ∈ class do
17:                 iterators.append(ANTERIOR(e))
18:             end for
19:         else
20:             ▷ otherwise append the closure
21:             rep ← REPRESENTATIVE(class)
22:             iterators.append(CLOSURE(rep))
23:         end if
24:     end for
25:     return iterators
26: end procedure
```

This partitioning is not performed in parallel - currently SOUFFLÉ operates on the expectation that the list returned is a STL container (and as such isn't embarrassingly parallel to work on). It would not be too difficult to modify this to accept our concurrent list to be returned, and thus be made to work in parallel. We believe the advantage for this approach would be most noticeable for many small disjoint-sets, or a single large disjoint-set (where we iterate over to generate `ANTERIOR` iterators).

3.1.5 Benchmarks

We perform simple benchmarks to both the worst- and best-case scenarios for the data-structure. We test two Datalog programs, one that generates a n-sized disjoint set, where the other generates n 1-sized disjoint sets. We compare the implementation against an explicit representation for this dataset, which uses a Binary Tree (BTree) as its underlying data-structure. This BTree is a heavily optimised thread-safe implementation, which is used as the default data-structure for all of SOUFFLÉ 's relations. These benchmarks were repeated 20 times for each run.

3.1.5.1 Large Disjoint Set

For the single disjoint-set program, the version that uses our new data-structure is demonstrated in Snippet 3.2. We mark the relation `mega` as an equivalence relation by putting the keyword `eqrel` as part of the declaration. This will now make the `mega` relation store its tuples within this equivalence relation data-structure. We use the term *eqrel* in experiments and figures to denote the implicit representation throughout the article.

The program is simple, we generate numbers from 1 until `lim(x)` (this is specified in a separate fact file), and also the numbers `lim(x) + 1` up until `lim2(y)` (also specified in a fact file). The `.printsize mega` statement will print the size of the relation (i.e. how many tuples are stored) at the end of the program.

LISTING 3.2: Single Set Datalog Eqrel Program

```
1  .decl gen1(x: number)
2  gen1(1).
3  gen1(x+1) :- gen1(x), !lim1(x).
4
5  .decl gen2(x: number)
6  gen2(x+1) :- gen2(x), !lim2(x).
7
8  .decl lim1(x : number)
9  .decl lim2(x : number)
10
11 .decl mega(x : number, y : number) eqrel
12 mega(x,y) :- gen1(x), gen2(y).
```

```
13
14 .input lim2()
15 .input lim()
16 .input gen2()
17
18 .printsize mega
```

If we specify `lim1(4)`, `gen2(5)`, and `lim2(8)`, this will result in the following facts:

gen1 → `gen1(1)`, `gen1(2)`, `gen1(3)`, `gen1(4)`

gen2 → `gen2(5)`, `gen2(6)`, `gen2(7)`, `gen2(8)`

This will allow the direct generation of `mega(1,5), mega(1,6), ..., mega(2,5), mega(2,6), ..., mega(4,8)`. Additionally, as the rule has been marked as an equivalence relation, then all the reflexive (e.g. `mega(1,1)`), symmetric (`mega(5,1)`), and transitive rules will be added in - however this example will not generate any rules that can exclusively be discovered through transitivity.

Snippet 3.3 is the equivalent explicit representation. Note the omitted `eqrel` specifier, and the additional rules to denote reflexivity, symmetry, and transitivity. For brevity, we omit the number generator relations.

LISTING 3.3: Single Set Datalog Explicit Program

```
1 .decl mega_explicit(x : number, y : number)
2 mega_explicit(x,y) :- gen1(x), gen2(y).
3 mega_explicit(x,x) :- mega_explicit(x, _).      // reflexive
4 mega_explicit(x,y) :- mega_explicit(y, x).      // symmetric
5 mega_explicit(x,z) :- mega_explicit(x, y), mega_explicit(y, z). //
    transitive
6
7 .printsize mega_explicit
```

We observe the running times for the ‘worst-case’ scenario in Figure 3.8.

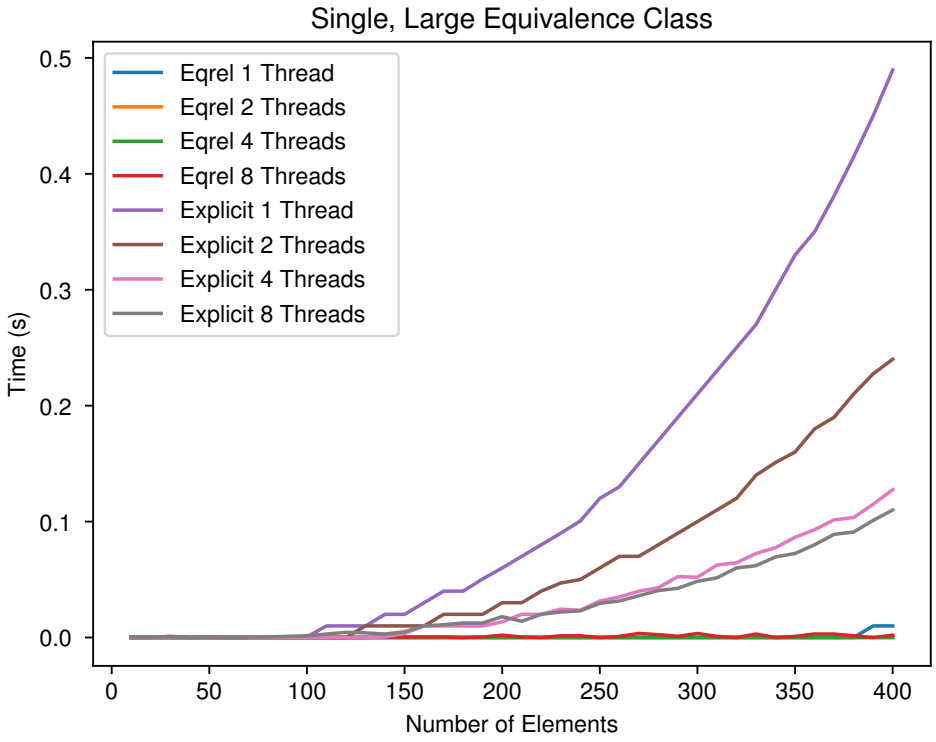


FIGURE 3.8: Total running time for a large equivalence class

The running times for the implicit representation programs appear to grow far too slowly to be measured accurately, for this small dataset. On the other hand, we are forced to restrict the experiment to run for lower input ranges due to the apparent quadratic increase in running time for the explicit program.

We can see the growth in memory appears to be linear in both programs. Whilst the trend for the explicit program shows a much steeper slope, this is not quadratic as would be expected. The data is much too erratic to be reliably measured however, as Figure 3.9 demonstrates.

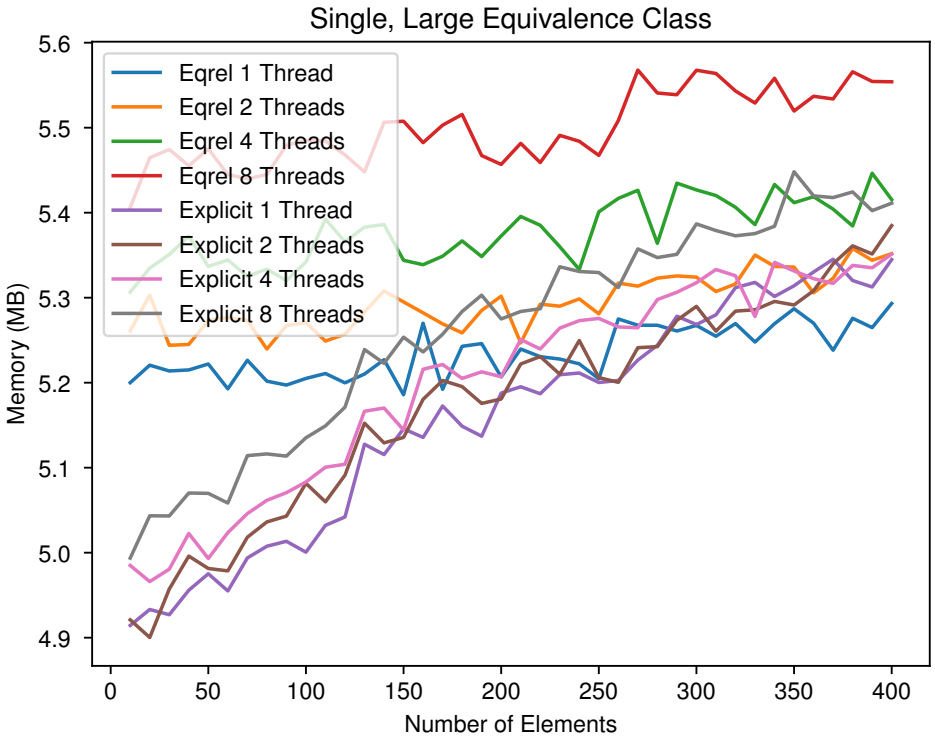


FIGURE 3.9: Total resident memory for a large equivalence class

We look at larger data-sets in section 4.1, where the expected quadratic memory consumption is observed. As a result of, we have empirically shown using an explicit representation may incur a quadratic space penalty for equivalence relations, while the implicit representation retains linear space.

3.1.5.2 Many Small Disjoint Sets

We also generate a benchmark which focuses on an example where there are no implicit pairs being stored. Instead of generating two series of numbers and performing a cross-product, we generate a single range, with the rule definition describing reflexivity only. We demonstrate the implicit eqrel program in Snippet 3.4, and the explicit program in 3.5. We omit the definition of `gen1`.

LISTING 3.4: Many Set Datalog Implicit Program

```
1 .decl mega(x : number, y : number) eqrel
2 mega(x,x) :- gen1(x).
```

The explicit version is similar to as is the case above, the only change is the initial definition of the rule to not include two ranges. No rule is needed to express reflexivity, as it is implied by the first rule of `mega_explicit`.

LISTING 3.5: Many Set Datalog Explicit Program

```
1 .decl mega_explicit(x : number, y : number)
2 mega_explicit(x,x) :- gen1(x).
3 mega_explicit(x,y) :- mega_explicit(y, x).
4 mega_explicit(x,z) :- mega_explicit(x, y), mega_explicit(y, z).
```

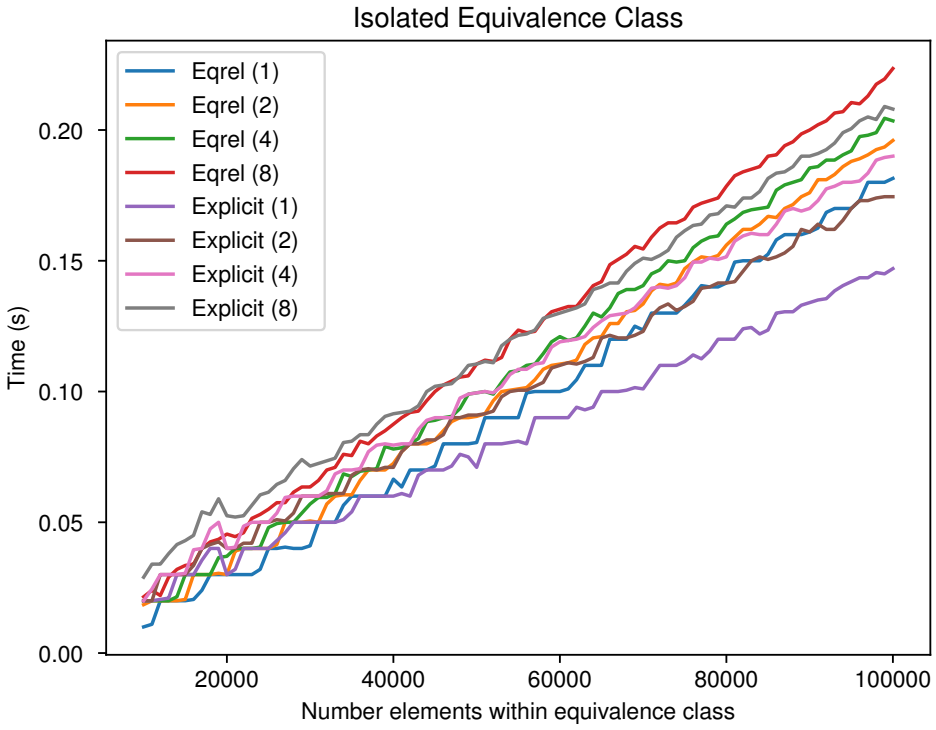


FIGURE 3.10: Total running time for small equivalence classes

As expected, the solving time for the implicit equivalence relation program is slower than the explicit representation. This test captures what would be the worst-case scenario with regards to solving time, as there is no additional pairs stored through the implicit equivalence relation. Interestingly, the overhead in runtime is only minor. For the implicit representation, we attribute this overhead to the cost of managing the auxiliary data-structures, such as the sparse mapping, cache, and more.

We see that for both versions, the runtime *increases* as the parallelism factor increases. We are currently unaware of the root cause or bottlenecks for this benchmark.

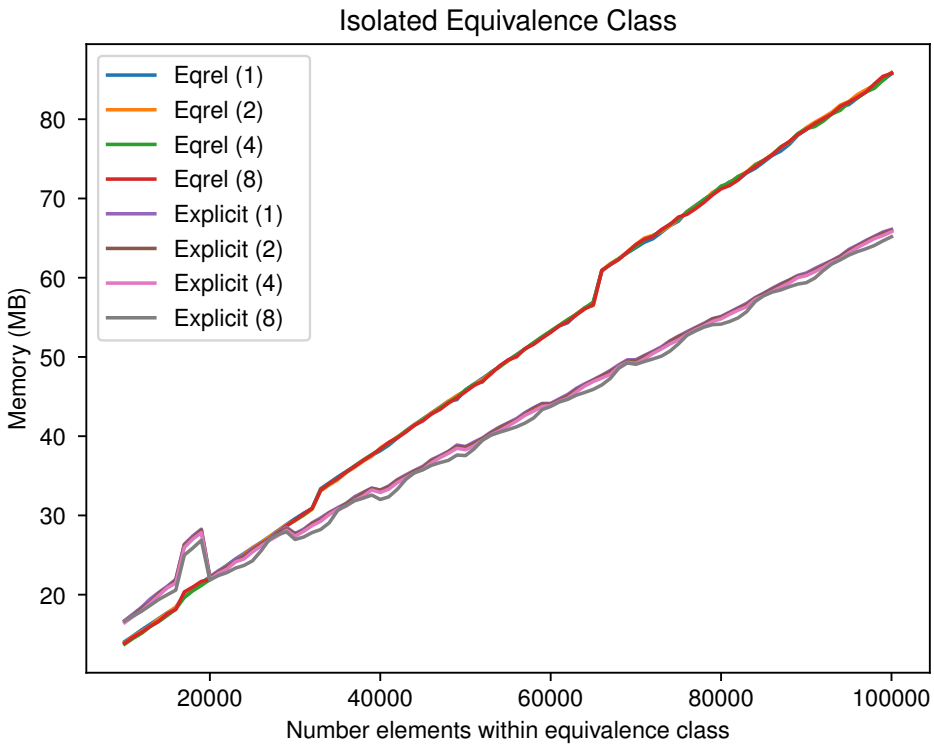


FIGURE 3.11: Total resident memory for small equivalence classes

We also see that the memory overhead for the implicit program scales poorly as compared to the explicit representation. We can attribute this to the multiple necessary hash-maps (equivalence cache, and Densifier layer (as described later)), and other auxiliary data that requires storage, although this would require further fine-grained profiling to confidently confirm.

Computational overhead from partitioning would also be a key contributor to the time slowdown observed in the implicit representation. This is especially the case for the isolated equivalence classes, as a separate iterator is generated for each of the classes which in this program is linear to the number of inputs. As part of the generated C++ code, SOUFFLÉ will always call this partitioning, even if there is no threading enabled (via compile-time flags). It would be worth investigating the speed-ups that may potentially be gained if this is omitted from the generated code for single-threaded execution. Additionally, as noted above, the partition function may be a candidate for parallelism - this would require additional internal profiling to investigate the potential benefits.

Another optimisation that should be benchmarked/profiled in the future, would be different conditions for partitioning, and the effect that they would impart on the time of iteration. Other partitioning approaches may result in a more balanced partitioning, and thus can reliably lead to better iteration performance, at the cost of partitioning time.

Investigating the bottlenecks of cache generation and reducing the number of lists that are invalidated during modification is another potential area of improvement for this Equivalence Relation data-structure layer.

3.2 Densifier

The union-find implementation of the lower layer uses a contiguous array to represent the forest of disjoint-sets efficiently. As a result, the indices for the array become the element's identifiers, that is, they are densely encoded. However, elements of input the equivalence domain are not necessarily tightly encoded, i.e., there could be gaps between two elements in its bitwise representation, depending on the program. As such, we require a way to store these arbitrary inputs in the relation.

To achieve a dense encoding of the elements, we resort to the notion of ordinal sets. An ordinal set is a set with a mapping between elements of the set and natural numbers, hence, the ordinal set of elements imposes a total order. The first element with respect to the total order corresponds to zero and the last element corrsponds to the cardinality of the domain minus one.

We name the equivalence domain $\mathbb{S}$. The ordinal set $\langle \mathbb{S}, o \rangle$ is defined by the carrier set $\mathbb{S}$ and the ordinal numbering function $o : \mathbb{S} \to \{0, \ldots, |\mathbb{S}| - 1\}$. The ordinal numbering function is bijective for which the inverse function $o^{-1}(l)$ is defined as $o(a) = l \Leftrightarrow o^{-1}(l) = a$.

The order among the elements in $\mathbb{S}$ is arbitrary, in practice these are unordered through the use of a hash-map to apply the forward mapping of o. This encoding of ordinal numbers is succinct: assuming that the ordinal number is provided, the elements of the ordinal set can be represented by $\log_2 |\mathbb{S}|$ elements. (Shannon, 1948)

For our implementation of disjoint-sets we require a densification of numbers, i.e., translating elements of the equivalence domain into their ordinal numbers.

We call this new data-structure the *Densifier*. The order among the elements is a temporal order. The first element that is added to the ordinal set is assigned zero; the second element one and so forth. Although this is a very simplistic data-structure, it requires high-performance for concurrent use.

3.2.1 ADT

For this purpose we construct an own-data structure that has three operations.

The first operation:

$$\langle \mathbb{S}', o' \rangle = \texttt{make\_element}(\langle \mathbb{S}, o \rangle, a)$$

takes the ordinal set $\langle \mathbb{S}, o \rangle$, and extends set $\mathbb{S}$ with element a and assigns a a new ordinal number in function o'. The resulting ordinal set is denoted by $\langle \mathbb{S}', o' \rangle$, and contains this new mapping.

The second operation:

$$\texttt{ordinal}(\langle \mathbb{S}, o \rangle, a) \Rightarrow l$$

maps element a to its ordinal number l i.e., the return value of `ordinal` is $o(a)$.

The third operation:

$$\texttt{sparse}(\langle \mathbb{S}, o \rangle, l) \Rightarrow a$$

maps an ordinal number l from the range $\{0, \ldots, |\mathbb{S}| - 1\}$ to its element a, i.e., the `sparse` operation returns the value $o^{-1}(l)$, the functional inverse of `ordinal`.

We implement the Densifier data-structure using a hash-map and a dynamic array. The hash-map is used to translate an element of the equivalence domain to an ordinal number and the array is used to map an ordinal number to an element. The issue of the implementation is that numerous threads will try to introduce new elements to the ordinal set. Thus it needs to be designed to avoid contention.

In our implementation, the `ordinal` and `make_element` operations are merged into a single interface - `densify` - where queries to retrieve the ordinal values for a given sparse value, either generate a new ordinal number and assign that to the sparse value iff the sparse value has not been seen before, otherwise the existing ordinal associated to the sparse value is returned.

The unification is performed due to the inherent relationship between the two - `make_element` shall not be called multiple times for a single sparse value, whilst `ordinal` must only be called for existing sparse values. This simplifies the implementation due to potential data-races if the methods were implemented separately.

Figure 3.12 simulates the following sequential ADT operations, where initially no sparse values have been queried. The return value of each query is the dense value pointed to for each sparse value.

`densify(a)` → `densify(b)` → `densify(c)` → `densify(a)` → `densify(d)`

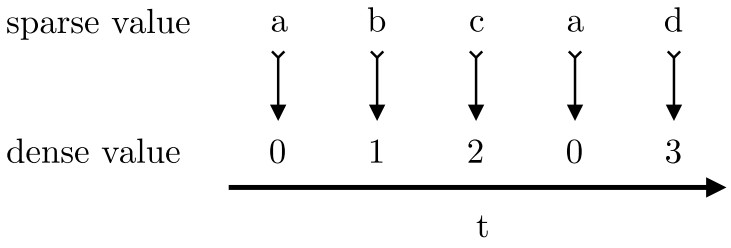


FIGURE 3.12: Simulation of densification of the sparse values $\{a, b, c, d\}$

This results in the following state in the hash-map, and dynamic array:

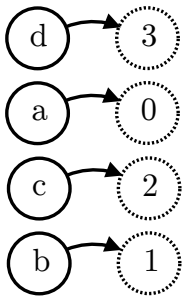


FIGURE 3.13: Resulting state in the hash map

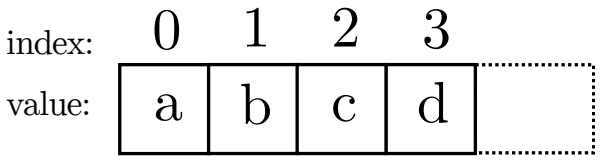


FIGURE 3.14: Resulting state in the dynamic array for the provided sequential operations.

To retrieve a dense value for a specified sparse value, we must only create it if it doesn't already exist within the mapping. In a concurrent context, we either may do this atomically via locking, or through optimistic allocation wherein a new value is created, atomically set in the hash-map if the key does not exist, otherwise the new value is cleaned up and the existing value is used. Either of these methods can be used, depending on the underlying hash-map. For example, Intel's `tbb::concurrent_hash_map` employs fine grained readers-writers locks, which abates the cost that coarse locking would incur.

For the latter method, we applied both the Libcuckoo hash-map (Goyal et al., 2018) and the Junction hash-map (Preshing, 2016a) with the optimistic approach.

3.2.1.1 Optimistic Assignment:

In order to minimise the number of clean up operations that occur, the look-up first checks whether there is a dense value already existing, and if so it is returned. If this is not the case, then a new ordinal value is assigned, which will then be attempted to be set as the dense value. We introduce the concept of a *getsert(x, a)* operation, which will attempt to set the value mapping for x to a iff it is not already set, and returns what the value currently is - if it had already been set, the original value is returned, otherwise the operation returns a. This *getsert* operation is analogous to a compare-and-swap operation, but in the context of hash-maps rather than the atomicity of single values. Checking this return value allows us to clean up our spuriously assigned ordinal value. Note that the `append(v)` operation doesn't just append v to the end of the array and return the index it was written to; the data-structure we use allows clean-up, that is, indices can be marked to be re-used, and the next `append` operation will use the next available free index.

We demonstrate the optimistic approach as `densify-opt` in Algorithm 9.

Algorithm 9 Obtain the ordinal value for this sparse key

```
1:  procedure DENSIFY-OPT(sparse)
2:      dense ← sparseToDenseMap[sparse]
3:      if dense does not exist then
4:          newDense ← denseToSparseMap.append(sparse)
5:          ▷ attempt to set the dense value to this newly created value
6:          dense ← sparseToDenseMap.getsert(sparse, newDense)
7:          if dense ≠ newDense then                          ▷ We were beaten
8:              allow newDense to be reassigned in the denseToSparseMap
9:          end if
10:     end if
11:     return dense
12: end procedure
```

This method of operation leaves little overhead in the range of the dense value domain. If there are t threads at most concurrent operating, and the number of elements in the sparse value set is n, our upper-bound for the largest element in the dense value domain is $n + t - 1$. For the typical execution contexts that SOUFFLÉ is used in, t is typically between 1 and 64, and that the largest value in the dense

value domain is observed to usually be $n - 1$ indicating low levels of contention on the last allocated elements.

Whilst dense values are assigned *to* sparse values, the converse is not true. We will only ever query dense values that already have an existing sparse value mapping to them (we insert the sparse value into the dense map first), and thus do not need to consider this event. As a result, retrieving a sparse value for a given dense value is trivial, simply return the value stored at index *dense*.

Algorithm 10 Obtain the sparse value for this ordinal key

```
1: procedure INVERSE(dense)
2:     return denseToSparseMap.at(dense)
3: end procedure
```

These algorithms do not strictly apply to all choices of data-structures that a sparse to dense map could be, as this `getsert(x,a)` operation may not necessarily be able to be implemented in a concurrent context. This operation may be replaced with a fine-grained locking scheme, which then allows us to eschew the overhead in the dense value domain, and keep it strictly tight. In Section 3.2.2, we explore the different requirements of the hash-maps.

3.2.1.2 Locking Assignment:

We also explore the second approach to keep consistency between the two mappings - the use of locks. A typical goal of locks is to ensure that each lock is not highly contended, which is the dominating cost of use. (Preshing, 2011) There are several methods for doing so; increasing the granularity to be fine grained locking, increasing the number of locks and *stratifying* over them, and more. The ability to use these different locking techniques is dependent on the use case.

We introduce stratified locking as a general technique to reducing lock contention, wherein multiple locks are created, and for an operation on an element that is required to be exclusive for that element, we deterministically choose a lock to obtain. If we are operating on a key-value store, we can use the key to determine which lock to obtain - by using the last $log_2(\#locks)$ bits of the key (or if it is not well distributed, of its hash) we can pick a lock by an index. For example, Snippet 3.6 demonstrates a simple stratified locking scheme, for 8 locks. We use a bitmask to select the index of which lock we will choose, and lock it. Unlocking would involve a similar lookup.

LISTING 3.6: Example Stratified Locking Scheme

```
1 void lock(K key) {
2    // assuming 8 locks
3    return locks[hash(key) & 0b111].lock();
4 }
```

This approach is general in that for any hash-map that is able to use the above optimistic allocation method, it can be transformed into a stratified locking example. We have not yet implemented this method, and as such do not yet know of the performance differences between both implementations.

We can easily use fine-grained locking; the Intel hash-map supports per-element locking, so that we can query if a dense value has been assigned and create a dense value for it atomically. This requires a large number of locks, and will lead to a linear space overhead, as each bucket requires its own lock, and the number of buckets is linearly proportional to the number of elements.

As we employ locking, the algorithm differs slightly, as seen in Algorithm 11 as `densify-tbb`. Although in the algorithm we explicitly unlock, in our implementation we don't require this; the writer's lock is implemented such that when the variable leaves its scope, the lock is automatically freed.

Algorithm 11 Obtain the ordinal value for this sparse key using a Intel TBB hash map

```
1:  procedure DENSIFY-TBB(sparse)
2:      writerLock ← empty reference
3:      ▷ access the element, and claim the writer's lock at that position
4:      isNew = sparseToDenseMap.find(sparse, writerLock)
5:      if isNew then
6:          ▷ reserve a new dense value, and assign it to this element
7:          newDense ← denseToSparseMap.append(sparse)
8:          writerLock.value ← newDense
9:      end if
10:     unlock writerLock
11:     return writerLock.value
12: end procedure
```

We have found through internal profiling that there was no significant benefit to using a reader's lock and upgrading to a writer's lock in the above algorithm, even for heavy read-only usage. In 3.2.3 we demonstrate the difference in performance for various usages.

3.2.2 C++ Implementation

The implementation of this Densifier layer acts as a FaÃğade for the below layer. It provides access to the Disjoint Set functionality, however the interface only deals with sparse values. For each Disjoint-Set ADT function we map to a dense value and calls the respective function on the disjoint-set layer, providing the dense value as an argument instead.

Retrieving a sparse value for a given dense value is required only for internal operations, specifically it is used for the fast generation of the equivalence cache, as mentioned in the Equivalence Relation section. This is primarily due to the constant overhead of densification via the hash-map; iterating over the keys, and finding the sparse representative is costly as compared to iterating over the contiguous disjoint-set array, and then performing sparsifcation via lookups in an array. As mentioned, we use a concurrent hash-map for the sparse to dense mapping, and a concurrent list ($\mathcal{O}(1)$ random access, this is described further in 3.3.3 as PiggyList).

We provide the following densification strategies for four different hash-maps:

(1) The STL `std::unordered_map` (GCC 7.3.0)
(2) Intel's `tbb::concurrent_hash_map` (Intel, 2017)
(3) Jeff Preshing's `junction::ConcurrentMap_Leapfrog` (Preshing, 2016b,a)
(4) `libcuckoo` by Goyal et al. (Li et al., 2014; Fan et al., 2013; Goyal et al., 2018)

The first, `std::unordered_map`, requires a coarse lock, as it supports at most one writer at a time. The Intel map as mentioned uses the fine-grained locking mechanism, whilst the last two (Libcuckoo and Junction) are both integrated using the optimistic allocation method. In the future we wish to test the stratified locking methods.

3.2.3 Benchmarks

In order to evaluate the performance of the Densifier with differing underlying data-structures, we run a series of benchmarks on several scenarios. As the dense to sparse mapping (sparsifcation) remains near equivalent between the different choices in hash maps, we only are required to benchmark the sparse to dense map (densification) implementations. Namely, we run `toDense(x)` for different sets of x, and degrees of concurrency.

We have four sets of choices of variables, for t threads, and n number of operations:

- **Same:** all threads will call `toDense(1)`
- **Unique:** all threads will call `toDense(i)` where i is within $\{1, \ldots, n\}$, and is uniquely densified once
- **Random:** all threads will call `toDense(i)` for some randomly chosen set of $i \in Z_{2^{32}}$
- **Contending:** all threads will call `toDense(i)` for $i \in \{1, \ldots, \frac{n}{t}\}$

The random set of variables is generated a priori to insertion, using the Xoroshiro128+ (Blackman and Vigna, 2018) random number generator. We intentionally choose a fixed domain to increase the relative level of contention within the densification calls. The final variable set attempts to ensure that each thread will be concurrently densifying the same sparse value & thus increase intention.

The benchmarks were performed on an 8 threaded machine with a Intel(R) Core(TM) i7-7700K CPU @ 4.20GHz CPU, with 64GB DDR4 RAM clocked at a frequency of 2333MHz.

3.2.3.1 Same-key densification:

Due to an unresolved memory leak in the Junction hash-map, we were forced to restrict it to lower element counts during benchmarking.

For single-threaded use, it is clear that the `std::unordered_map` is significantly faster, although as the parallelism factor the STL container degrades drastically. Interestingly, the Libcuckoo map performs best in the 8 threaded environment, but is worse than TBB in 4 threads, and also marginally for 2 threads. Online benchmarks tend to show that Junction significantly better (Preshing, 2016c), yet we observe the contrary across most benchmarks.

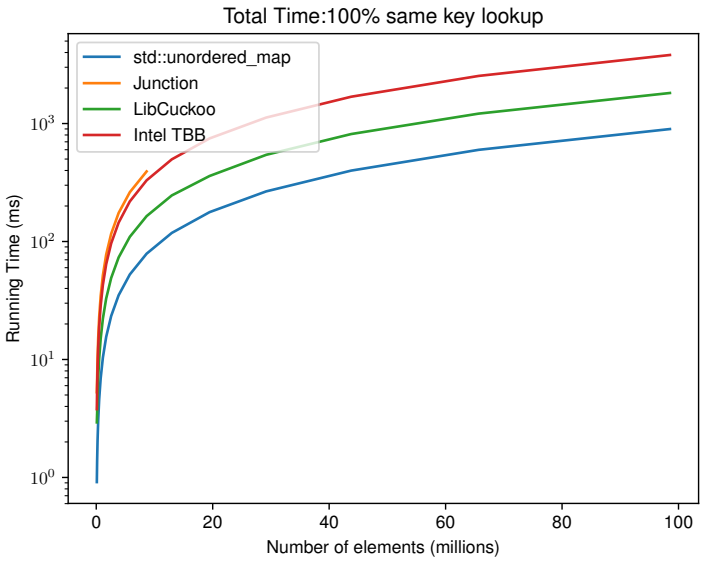


FIGURE 3.15: Same key densification, single thread

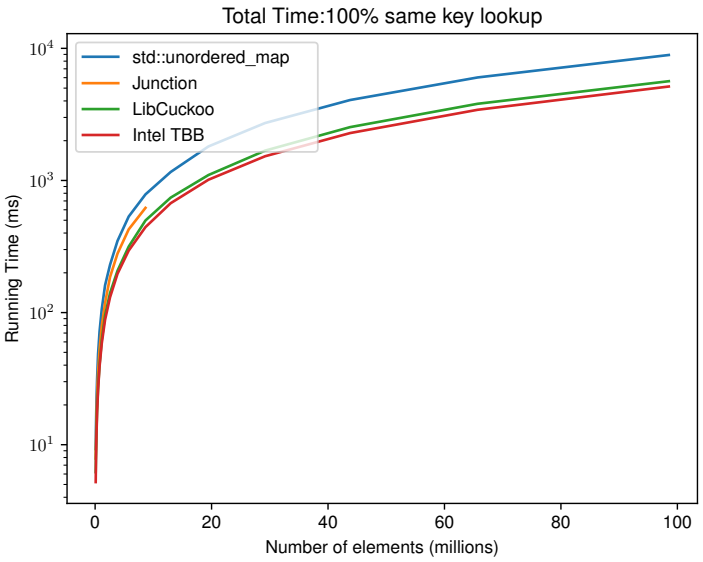


FIGURE 3.16: Same key densification, two threads

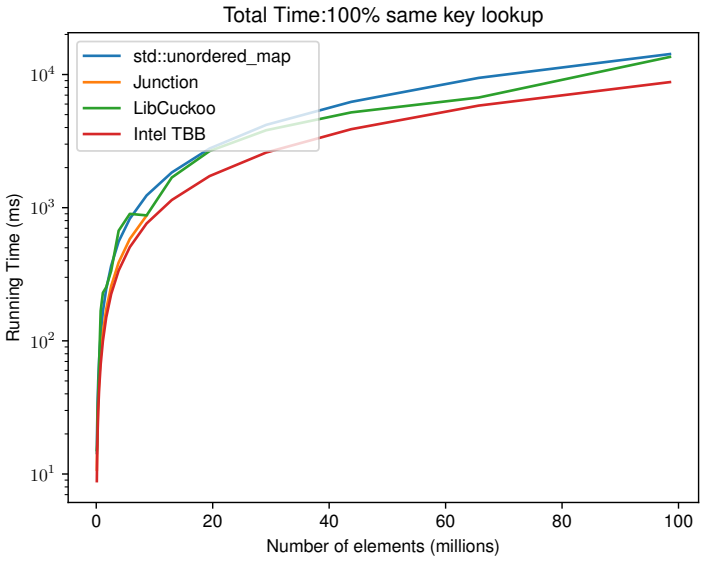


FIGURE 3.17: Same key densification, four threads

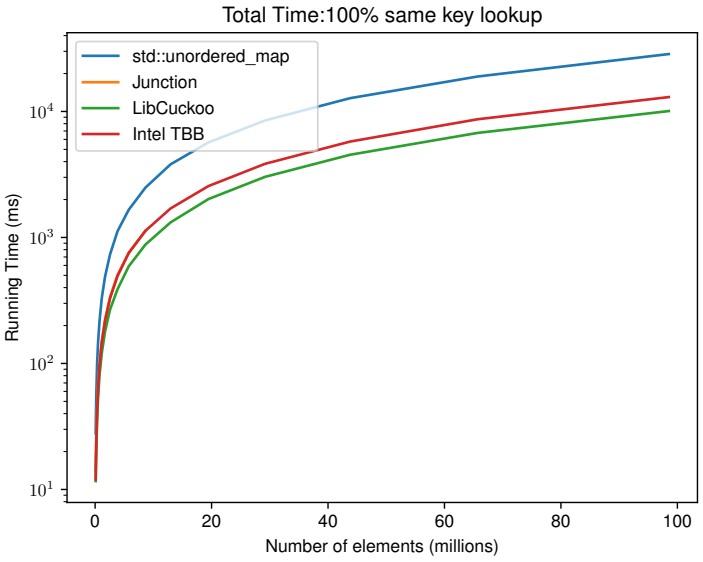


FIGURE 3.18: Same key densification, eight threads

3.2.3.2 Unique-key densification:

We see a similar result for single-threaded as compared to same-key densification, however for unique keys, Junction performs better than the Intel hash-map. Libcuckoo performs consistently better than the Intel map for all number of threads. This is interesting, as for this example, there is no lock contention in the Intel map, potentially due to the inherent nature of the hash-maps internals. This is most clear for the single-threaded run, the performance of the Intel hash-map is almost half the speed of the Libcuckoo map, and nearly 6 times slower than the `std::vector`.

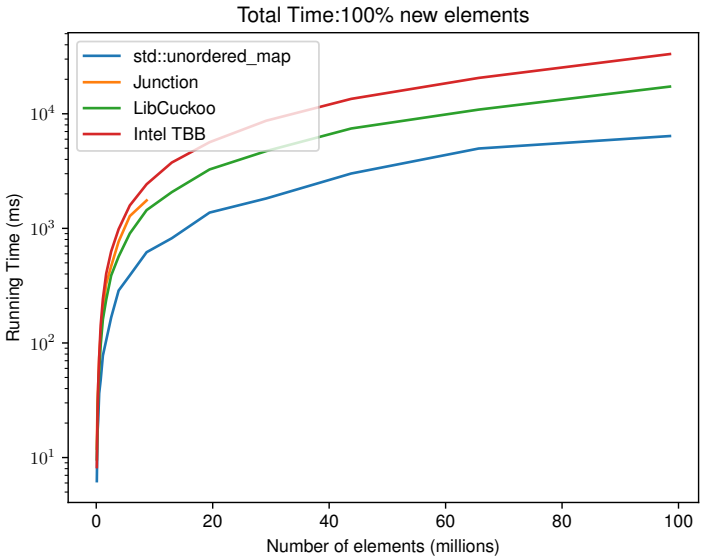


FIGURE 3.19: Unique key densification, single thread

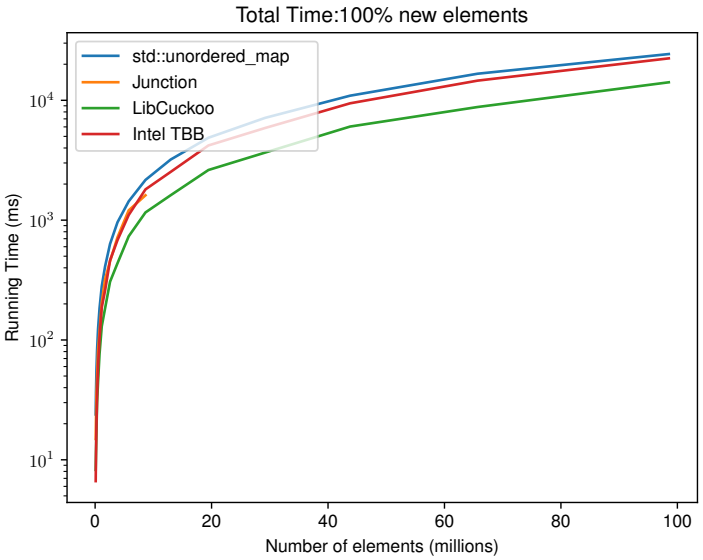


FIGURE 3.20: Unique key densification, two threads

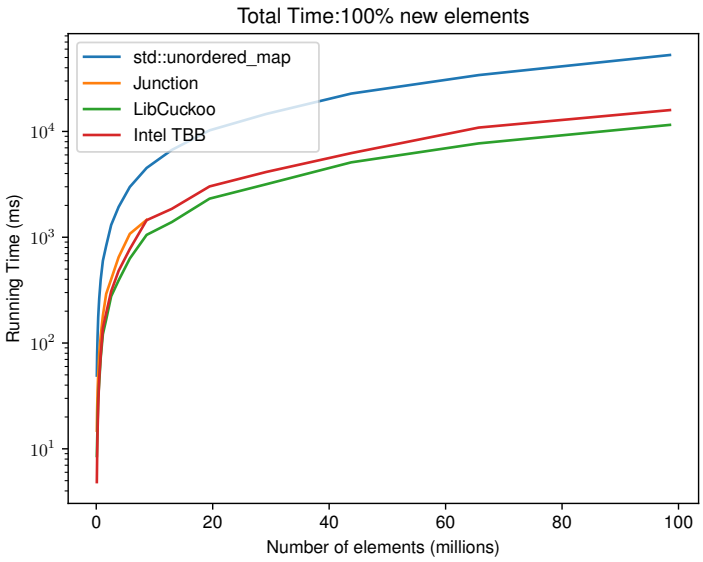


FIGURE 3.21: Unique key densification, four threads

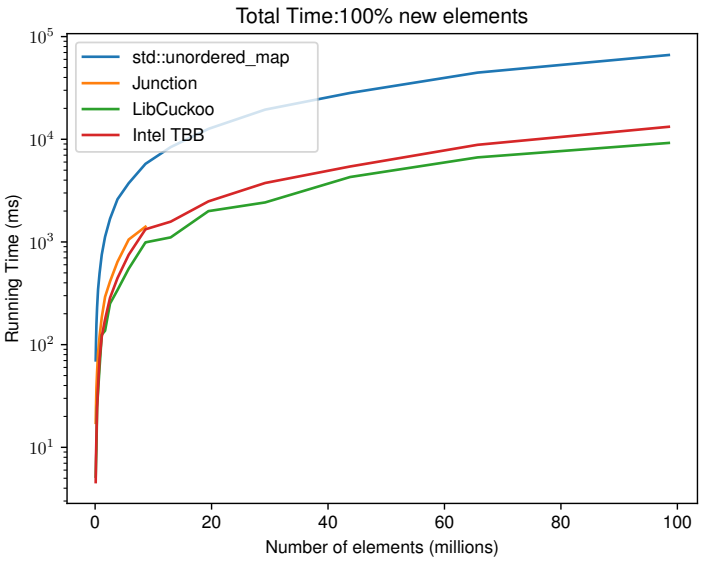


FIGURE 3.22: Unique key densification, eight threads

3.2.3.3 Random-key densification:

The differences between Libcuckoo and the Intel map are similar to the unique-key densification benchmark. Interestingly, Junction has improved performance as compared to the other maps for single- and two-threaded runs.

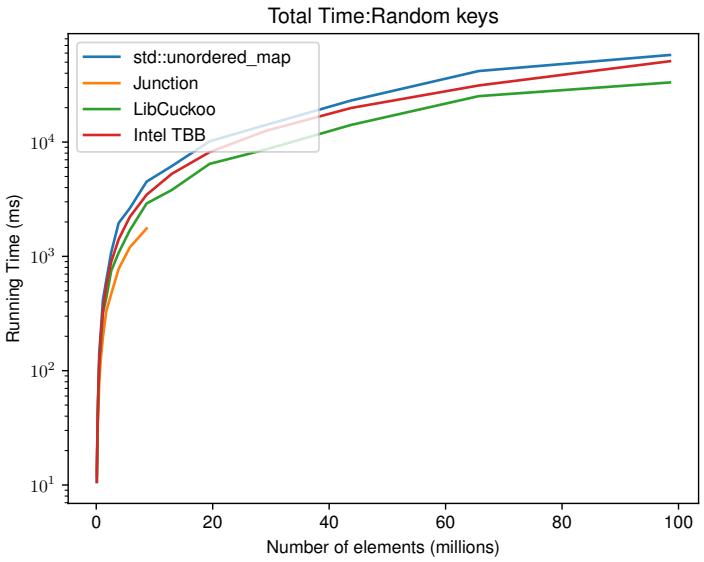


FIGURE 3.23: Random key densification, single thread

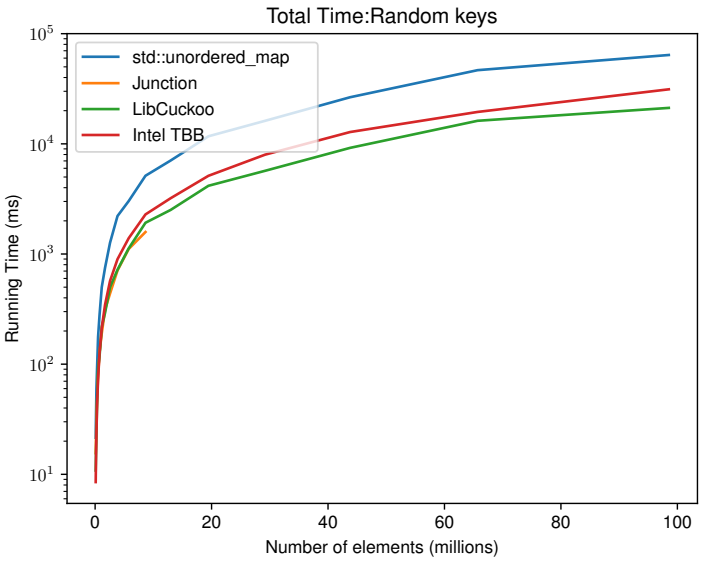


FIGURE 3.24: Random key densification, two threads

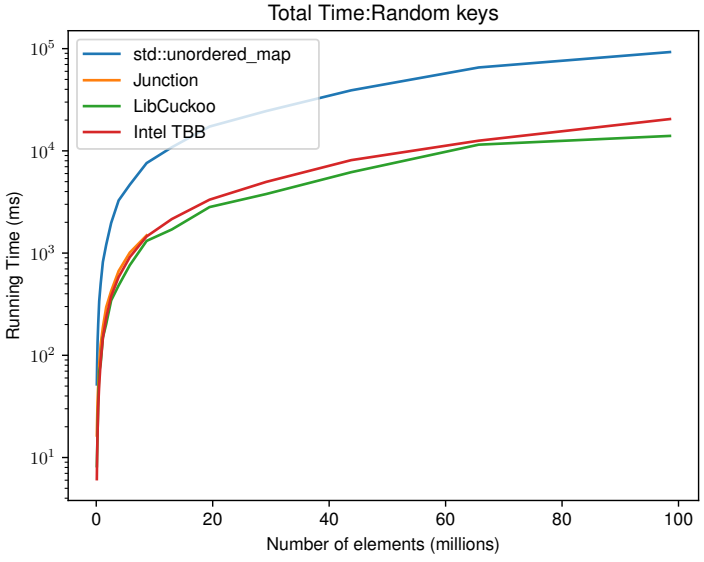


FIGURE 3.25: Random key densification, four threads

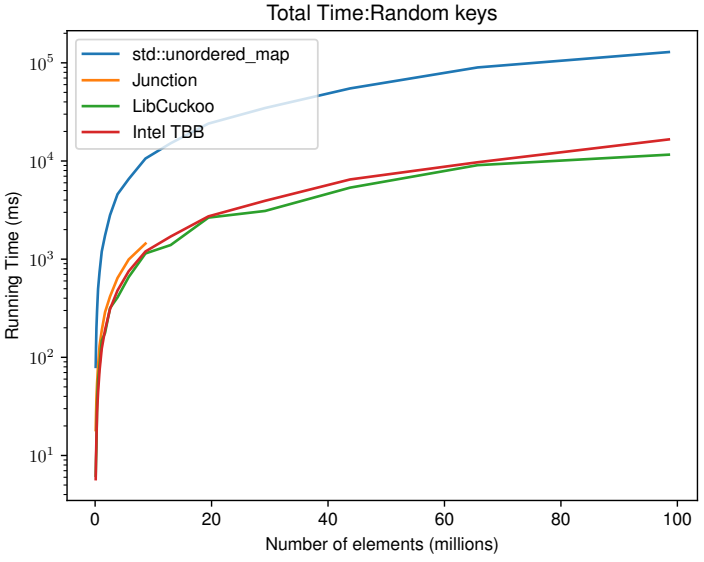


FIGURE 3.26: Random key densification, eight threads

3.2.3.4 High contention key densification:

In this benchmark, Intel's map performs better than Libcuckoo. This is interesting, as lock contention is regarded to be a major component to performance degradation, and this experiment is aimed to maximise this. In fact, the Intel hash-map performs nearly 40% faster, we are currently un-aware for the root cause for this.

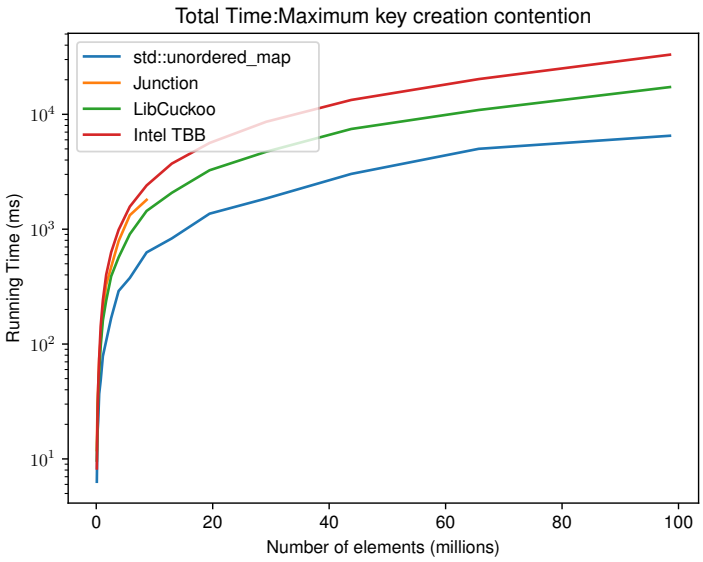


FIGURE 3.27: High contention key densification, single thread

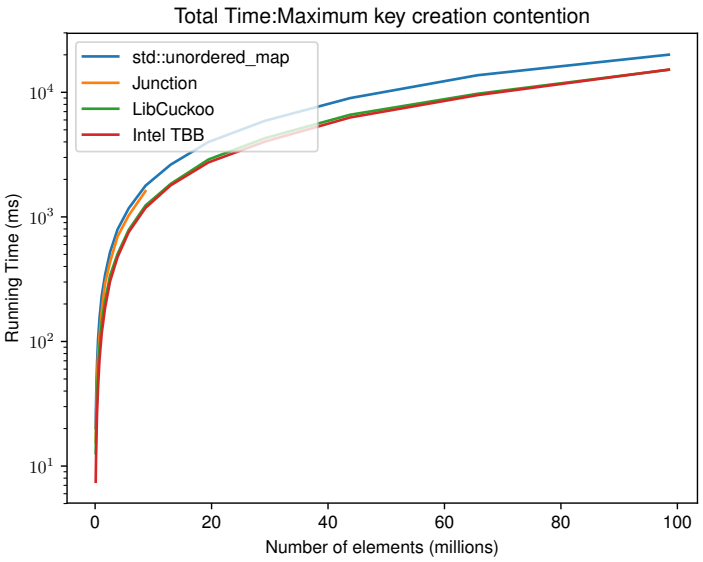


FIGURE 3.28: High contention key densification, two threads

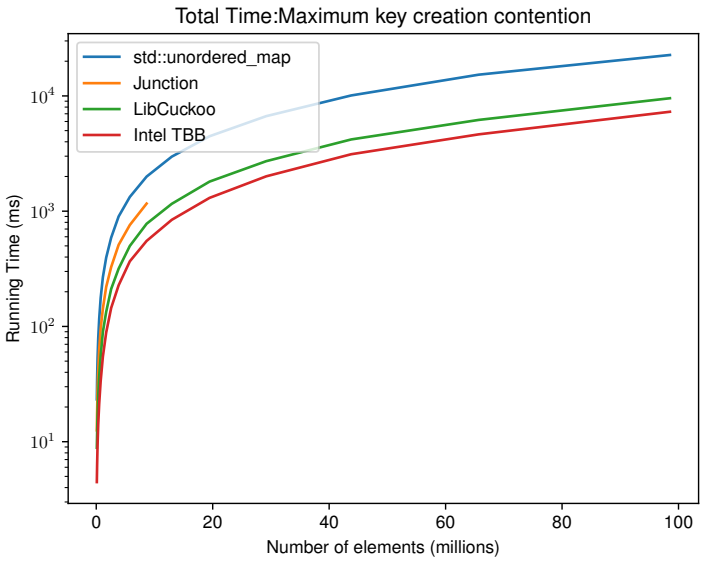


FIGURE 3.29: High contention key densification, four threads

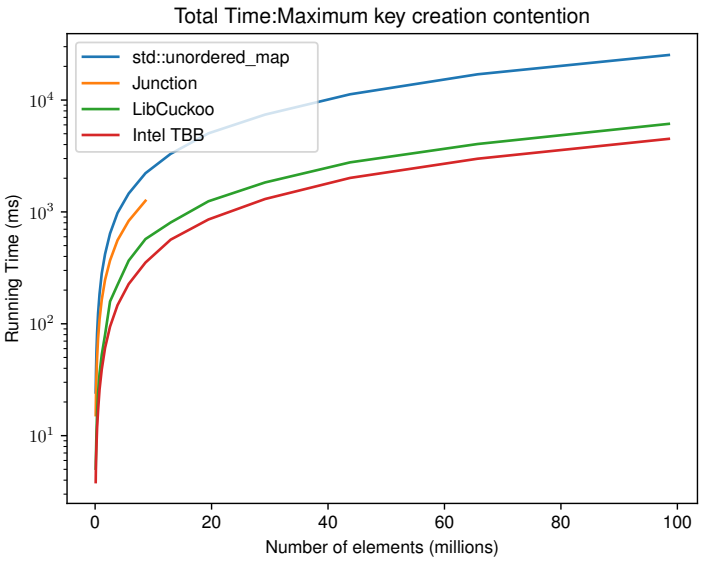


FIGURE 3.30: High contention key densification, eight threads

We observe a similar performance between the Intel and Libcuckoo maps, with the latter demonstrating superior performance in the majority of benchmarks, especially for single-threaded runs. The current implementation of the densification layer uses the Intel hash-map, due to some unresolved bugs in integrating the Libcuckoo hash-map.

3.3 Disjoint Set

Union-find is a very efficient data-structure to partition a set of elements D into disjoint-sets. Initially, all element reside in their own partition, i.e., equivalence classes which is also known as a singleton. The disjoint-set data-structure provides three operations, i.e., `make_set`, `union`, and `find`. The operations are performed on elements in the domain D. Note that in some implementations the domain D is not necessarily bound.

3.3.1 ADT

make_set(x): This operation creates a singleton for element x in the partitioning of set D, i.e., x resides in its on disjoint set. For example, `make_set` applied on elements w, x, y, z in sequence produces following partitionings of set D:

(1)
$$D : \emptyset \Rightarrow \texttt{make\_set(w)} \Rightarrow D : \{\{w\}\}$$

(2)
$$D : \{\{w\}\} \Rightarrow \texttt{make\_set(x)} \Rightarrow D : \{\{w\}, \{x\}\}$$

(3)
$$D : \{\{w\}, \{x\}\} \Rightarrow \texttt{make\_set(y)} \Rightarrow D : \{\{w\}, \{x\}, \{y\}\}$$

(4)
$$D : \{\{w\}, \{x\}, \{y\}\} \Rightarrow \texttt{make\_set(z)} \Rightarrow D : \{\{w\}, \{x\}, \{y\}, \{z\}\}$$

union(x,y): Merges the two disjoint sets containing the elements x and y. Note if both elements reside in the same disjoint set of the partitioning then the partitioning of the set D will not change after the union-operation. For example, let's assume a partitioning of D is $\{\{w\}, \{x\}, \{y\}, \{z\}\}$, observe the following operations:

(1)
$$D : \{\{w\}, \{x\}, \{y\}, \{z\}\} \Rightarrow \texttt{union(w,x)} \Rightarrow D : \{\{w, x\}, \{y\}, \{z\}\}$$

(2)
$$D : \{\{w, x\}, \{y\}, \{z\}\} \Rightarrow \texttt{union(w,y)} \Rightarrow D : \{\{w, x, y\}, \{z\}\}$$

find(x): Retrieve the *representative* of a disjoint set in the partitioning. A disjoint set has exactly one representative. Iff two elements in D have the same representative, they belong in the same disjoint set. For example, assume a partitioning $D : \{\{w, x, y\}, \{z\}\}$. We choose following elements as the representatives of the disjoint sets x is the representative of the disjoint set containing exactly w, x, y, whilst z is the representative of its own set; singletons always are their own representative. So, for the above partitioning, observe the following facts:

$$find(w) = find(x) = find(y)$$

$$find(z) \neq find(x)$$

With the disjoint-set data-structure we can represent equivalence-relations implicitly. A disjoint set in the partitioning forms a class in the equivalence relation. Assume that $D = D_1 \cup ...D_2$ is a partitioning of domain D, i.e., for all i, j where $i \neq j$, sets $D_i \cap D_j = \emptyset$. An equivalence relation $R \subseteq D \times D$ induces a partitioning of D where the disjoint sets correspond to the equivalence classes of R, i.e.,

$$\forall i : \forall a, b \in D_i : a \, R \, b$$

The disjoint-set representation of an equivalence class is condensed. The worst-case space complexity of an equivalence relation R is given by $\mathcal{O}(|D|^2)$ assuming we store all possible pairs $(a, b) \in R$ in memory. This worst-case space complexity is tight in case if all elements in D relate to each other, i.e., $\forall a, b \in D : aRb$. The best-case space complexity is $\mathcal{O}(|D|)$, if each element in R only relates to itself, i.e., all equivalence classes in R form singletons.

However, by representing an equivalence relation by its equivalence classes and its induced partitioning on the domain D, the space complexity is in the order of the number of elements in the domain, i.e., $\mathcal{O}(|D|)$. Hence, the disjoint-set representation of an equivalence class is superior to an explicit representation storing each pair separately.

An efficient implementation (Galler and Fisher, 1964) of disjoint sets utilises a collection of trees, each representing a disjoint set to comprise a disjoint set forest; unioning elements would be performed by joining trees, and the root of the tree can be considered to be the representative for that set. There are several optimisations to achieve a near-constant amortised complexity.

The naïve approach for unioning a and b is to attach trees by drawing an edge from the root of b's tree, to a's root, as demonstrated for an initial partitioning of $D : \{\{w\}, \{x\}, \{y\}, \{z\}\}$,

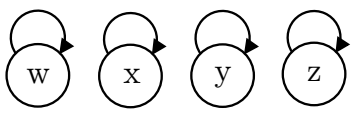


FIGURE 3.31: Pre-union

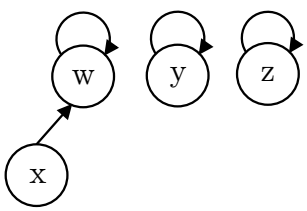


FIGURE 3.32: After `union(w,x)`

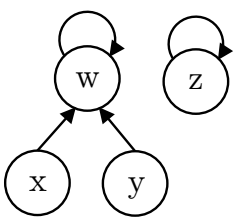


FIGURE 3.33: After `union(w, y)`

This has a worst case tree height of n, considering the scenario wherein the larger tree is attached to a new singleton repeatedly, leading to a $\mathcal{O}(n)$ cost for `find(x)`.

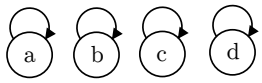


FIGURE 3.34: Pre-union

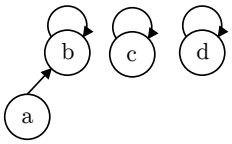


FIGURE 3.35: After `union(b,a)`

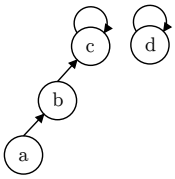


FIGURE 3.36: After `union(c, b)`

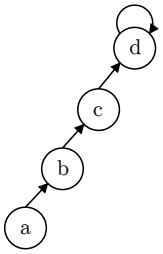


FIGURE 3.37: After `union(d, c)`

By associating a value with each root that is the number of nodes in that root's tree, `union(x,y)` operations can be altered to always attach the smaller tree to the larger one. On a tie, one is selected to be the leader, and the other attached. This results in a max height of $\lfloor log_2 n \rfloor$, thus `find(x)` and `union(x,y)` operations are now logarithmic. [M.D.McIlroy] For the initial carrier set, D : $\{\{a\},\{b\},\{c\},\{d\}\}$, in Figures 3.38, 3.39, 3.40, 3.41, observe the result of the following operations to the disjoint set forest. The value beside each node's label is the corresponding *weight* of that subtree, i.e. the number of nodes that are part of the subtree with the node as its root.

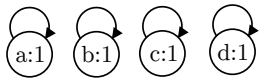


FIGURE 3.38: Pre-union

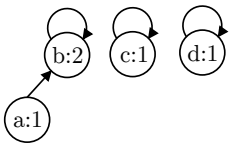


FIGURE 3.39: After `union(b,a)`

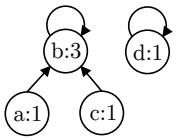


FIGURE 3.40: After `union(c, b)`

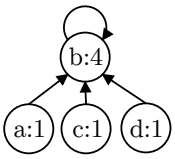


FIGURE 3.41: After `union(d, c)`

Furthermore, we can compress the paths as operations occur, such that they point to the resulting root. The mechanism for doing so, is that as we traverse up from a node x in order to find the root, we keep track of the root, and perform a second traversal, which then sets the parents of all nodes on that path from x to its root to now point directly to the root.

Path compression allows an even faster computational bound - instead of logarithmic, the `union(x,y)` and `find(x)` operations have an amortised complexity of $\mathcal{O}(\alpha(n))$, where $\alpha(n)$ is the inverse Ackermann function - one that grows incredibly slowly (for $n = 10^{80}$, $\alpha(n) \leq 4$).

In this case, instead of associating the size of the tree as a value to each root, we associate a value called the rank, now defined as $r(x) = h(x) - 1$, $h(x)$ being the height of the node x in the forest. This allows us to store the rank in $log_2 log_2(n)$ bits, (Shannon, 1948) which proves useful later in concisely storing the rank as described in Section 3.3.2.1.

For a given carrier set $D : \{\{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{f\}, \{g\}\}$, observe the following operations. The value next to each node is now not the number of elements in that node's subtree, but rather its rank.

FIGURE 3.42: Pre-union

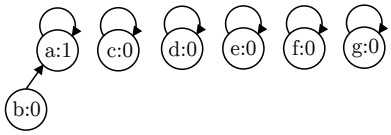


FIGURE 3.43: After `union(a,b)`

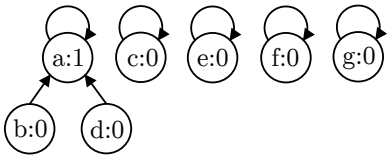


FIGURE 3.44: After `union(b,d)`

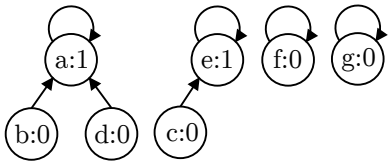


FIGURE 3.45: After `union(e,c)`

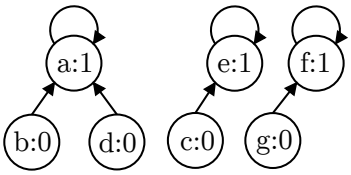


FIGURE 3.46: After `union(f,g)`

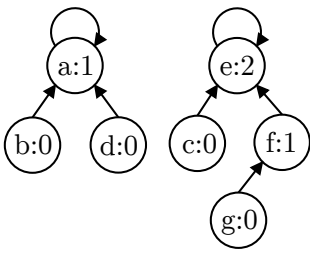


FIGURE 3.47: After `union(e,f)`

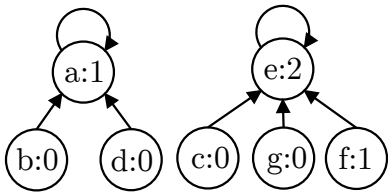


FIGURE 3.48: After `find(g)`

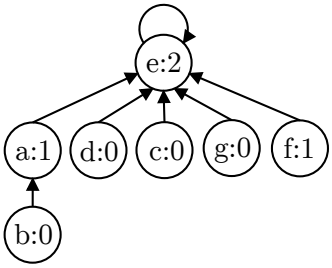


FIGURE 3.49: After `union(d,e)`

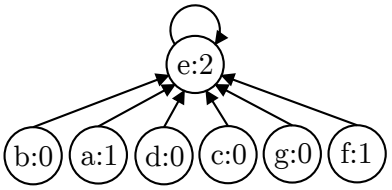


FIGURE 3.50: After `find(b)`

Anderson's 1991 wait-free parallel Union-Find data-structure (Anderson and Woll, 1991) provides a simple design on which to base the Disjoint Set layer of our data-structure on. As an underlying machine they provide one based on a Parallel Random-Access Machine (PRAM), which is a theoretical machine model for parallel computations. PRAM as defined by Wyllie (Wyllie, 1979) consists of some unbounded global memory (accessible and writable by all processors), an unbounded number of processors, and some input plus a program both being finite in size. Each processor also has unbounded local memory, an accumulator, a program counter, and a flag whether or not the processor is running; all exclusive, such that a processor cannot write into another processor's local memory. Anderson's model represents all instructions as atomic.

Anderson's data-structure is wait-free such that each thread will take a finite number of steps to complete - independent of other threads behaviour. Wait-free data-structures often require the use of primitives beyond simple atomic assignment. One such primitive that is required, is `COMPARE-AND-SWAP(x,a,b)`, as shown in Algorithm 12. This primitive is atomic, as dictated by Anderson's model; x is set to b if $x = a$.

Algorithm 12 Compare-and-swap primitive

```
1: procedure COMPARE-AND-SWAP(x, a, b)
2:     if x = a then
3:         x ← b
4:         return success
5:     end if
6:     return failure
7: end procedure
```

Anderson's data-structure uses an array $A[1..n]$ for representing the partitioning of the carrier set $D = \{1, \ldots, n\}$ assuming a fixed cardinality at the creation time of the data-structure. From now on, we consider the array $A[0, \ldots, n-1]$, to represent the carrier set $D = \{0, \ldots, n-1\}$, for ease of representation in C++ .

However, in Datalog we need to cope with a monotonically growing carrier set. Hence, the carrier set is initially an empty set which gets extended throughout its use.

To overcome the limitation of a fixed n in Anderson's data-structure, we extend the interface with an additional operation `NEW-ELEMENT()` that creates a new element in the carrier set; initially a singleton.

For this extension, we need to swap the static array in Anderson's data-structure with a dynamically increasing array called PiggyList. The PiggyList is an array that works with parallel threads accommodating for concurrent capacity expansions, concurrent reads, and concurrent writes, each performing well.

In the following, we provide an overview of Anderson's parallel disjoint set data-structure with our extensions. The operations are `FIND(x)`, `UNION(x,y)`, `SAME-SET(x,y)`, `NEW-ELEMENT()`.

`NEW-ELEMENT()` introduces a new singleton into the carrier-set; with regards to our disjoint-set forest represented in $A[0..n-1]$, this element has rank 0, and is the root of its own tree.

Algorithm 13 Method to introduce a new element into the carrier set

```
1: procedure NEW-ELEMENT
2:     new ← number of elements currently in A
3:     A[new] ← CREATE-RECORD()
4:     A[new].parent ← new
5:     A[new].rank ← 0
6: end procedure
```

The operation `FIND(x)` shown in Algorithm 14 finds a path from the node `x` to its root node and returns the root node as a representative of the disjoint set in which `x` resides in. As a side effect of finding the path, all nodes along the path will be attached to the root node, i.e., collapsing the path. In a parallel version, there may be an issue with a data race occuring while updating the parent node of an element. This is prevented by the `compare-and-swap` operation that ensures that no other thread has changed the parent node spuriously. This proves not to be an issue, as whether or not the compare-and-swap succeeds, *c-element* stores its grandparent, thus *c-element* is always updated to an ancestor element in its disjoint-set tree.

Algorithm 14 Method to retrieve the representative for a given element

```
1:  procedure FIND(x)
2:      c-element ← x
3:      while c-element ≠ A[c-element].parent do
4:          parent-el ← A[c-element].parent
5:          ▷ Attempt to update the current element's parent to its grandparent
6:          COMPARE-AND-SWAP(A[c-element].parent, parent-el, A[parent-el].parent)
7:          c-element ← A[parent-el].parent
8:      end while
9:      return c-element
10: end procedure
```

The operation `SAME-SET(x,y)`, described in Algorithm 15, is a predicate that holds, if the elements `x` and `y` reside in the same disjoint set; otherwise the predicate fails. The implementation of `SAME-SET(x,y)` uses the operation `FIND` for its implementation. First, it determines the representatives of `x` and `y` respectively. If they coincide, we return true. However, a concurrent thread may change the forest. To check this case, on line 8 we check whether `x`'s found rep after the find is still its own representative. If this is the case, the `SAME-SET(x,y)` predicate fails. In all the other cases, we have detected contention and we restart the operation again.

Algorithm 15 Predicate to test whether two elements are within the same disjoint set

```
1:  procedure SAME-SET(x,y)
2:  restart:
3:      xrep ← FIND(x)
4:      yrep ← FIND(y)
5:      if xrep = yrep then
6:          return true
7:      end if
8:      if A[xrep].parent = xrep then
9:          return false
10:     end if
11:     go to restart
12: end procedure
```

The operation `UNION(x,y)` merges the disjoint sets of elements `x` and `y`, as demonstrated in Algorithm 16. We have several cases. First, if the two elements reside in the same disjoint set, i.e., the representatives are the same, no updates of the data-structure are performed. Second, if the two elements reside in different disjoint sets, the trees of both disjoint sets are merged. The merge order is dictated by the rank, in that we attach the representative with a smaller rank to the other. In the case of a tie, we still attach the two, however we increment the rank of the second, `yrep`.

Algorithm 16 Method to merge two element's disjoint sets

```
1:  procedure UNION(x,y)
2:  restart:
3:      xrep ← FIND(x)
4:      yrep ← FIND(y)
5:      if xrep = yrep then
6:          return
7:      end if
8:      xreprank ← A[xrep].rank
9:      yreprank ← A[yrep].rank
10:     if xreprank > yreprank or (xreprank = yreprank and repx > repy) then
11:         SWAP(xrep,yrep)
12:         SWAP(xreprank, yreprank)
13:     end if
14:     if UPDATE-ROOT(xrep, xreprank, yrep, yreprank) = failure then
15:         go to restart
16:     end if
17:     if xreprank = yreprank then
18:         UPDATE-ROOT(yrep, yreprank, yrep, yreprank + 1)
19:     end if
20: end procedure
```

Used in the above methods, `UPDATE-ROOT(x, oldrank, y, newrank)` is a helper function, defined in Algorithm 17. Its behaviour is to set `x`'s parent to be `y`, with a new rank of `newrank`. This is only done if `x` is a root node (i.e. its parent is itself), and `x`'s record has not been concurrently modified since (this is performed by the `COMPARE-AND-SWAP`).

Algorithm 17 Wait-free method to update a tree root of the disjoint set forest

```
1:  procedure UPDATE-ROOT(x, oldrank, y, newrank)
2:      old ← A[x]
3:      if old.parent ≠ x or old.rank ≠ oldrank then
4:          return failure
5:      end if
6:      new ← CREATE-RECORD()
7:      new.parent ← y
8:      new.rank ← newrank
9:      return COMPARE-AND-SWAP(x, old, new)
10: end procedure
```

`SAME-SET` and `UNION` both have loops within their function bodies (jumps to `restart:`), and are affected by other threads' behaviour. Interestingly, this behaviour seems contradictory to the wait-free claim put forth by Anderson. The case can be made that they in fact will terminate in a finite number of steps, as whilst they may loop, progress is always made in each loop, so perpetutity requires continous introduction of elements. In Anderson's case, they fix the cardinality to a n, known ahead of time, so

the number of elements are fixed, and thus there are a finite number of function calls that will cause contention.

For `SAME-SET` and `UNION`, this may only happen if during each loop, the height of the disjoint-set tree gains a new root right before checking each iteration. In `UNION` this must occur somewhere between line 4 and line 14, and for `SAME-SET` it must happen between line 4 and line 8. For a new root to appear ahead of the found root, the disjoint-set tree currently being operated on must be attached to another, which only occurs if the rank of the other disjoint-set tree is greater than or equal to the current disjoint-set tree. For a larger rank to occur, the other tree must have at least the same number of elements within. As this must occur every loop, this mandates that the size of the current disjoint-set tree will double, due to the attachments.

We construct a similar argument to Anderson, that despite our added capability to insert new elements, the number of elements to be added is externally bound - Datalog only deals with finite domains (Abiteboul et al., 1995), and as such proves our extension upholds the wait-free property of Anderson's original data-structure for our use case.

3.3.2 Implementation

Anderson's description of the data-structure is based on PRAM, and thus requires modification for a C++ implementation. Introduced in C++11 is a standardised memory model, simplifying the implementation, yet still requiring synchronisation primitives to ensure intended memory consistency (Various, 2018).

Notably, in order to have reasonable performant atomic operations as required by the `COMPARE-AND-SWAP` primitive, we require the records to be at most 8 bytes. SOUFFLÉ currently supports 64bit machines, so larger records sizes, such as 16 bytes, require SSE instructions, which cannot be performed atomically without a significant performance hit, visualised in Figure 3.54. We require specialised encoding of these records to accommodate this, whilst also obeying storage requirements.

As previously mentioned, to support arbitrary sized carrier-sets, as they are not known ahead of time, we also make use of a data-structure called PiggyList.

3.3.2.1 Element Representation

We store our elements in a similar fashion that is described in Anderson's model. Elements are stored in an array, with each index holding a record that contains two fields, *parent* and *rank*. The value of the index represents the element's identifier, in Figure 3.51, 1 is the name and value of the element stored in index 1.

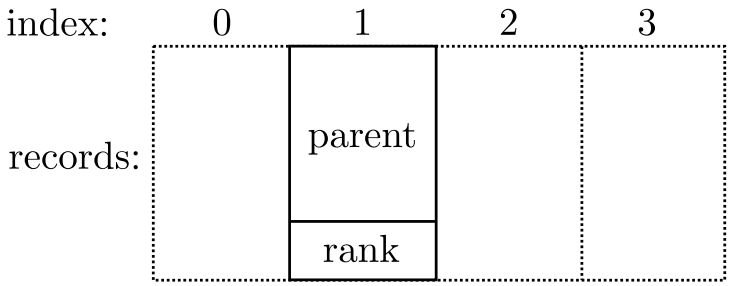


FIGURE 3.51: packed value of an element resident in index 1

The *parent* field stores the index of the element that has this element as a child, whilst *rank* stores the *quasi-height* of the element; disjoint-set forests undergo path compression, so this value is an over-approximation of the actual height. The array representation shown in Figure 3.52 corresponds to the disjoint-set forest as represented in Figure 3.53. A possible series of operations to have led to this state is `ADD-ELEMENT()`, `ADD-ELEMENT()`, `UNION(0,1)`, `ADD-ELEMENT()`.

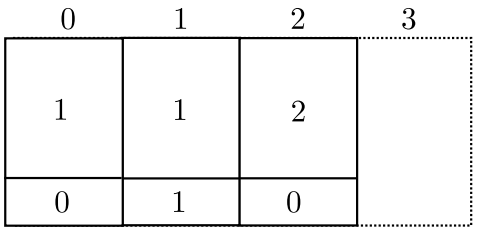


FIGURE 3.52: Array representation of packed values

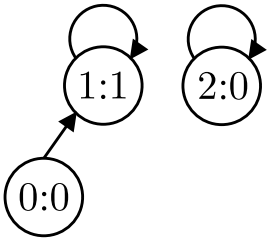


FIGURE 3.53: Equivalent disjoint-set forest

In our C++ implementation, the representation of the record was stored entirely in a 64 bit unsigned integer. We provide the following C++ type definitions.

```
1 typedef uint8_t rank_t; /* rank */
2 typedef uint64_t parent_t; /* parent */
```

```
typedef uint64_t block_t; /* record of both rank, parent */

// number of bits that the rank is
constexpr uint8_t split_size = 8u;
// block_t & rank_mask extracts the rank
constexpr block_t rank_mask = (1ul << split_size) - 1;
```

To pack and unpack the values to and from the record respectively, the following functions are specified. `b2p(block)` retrieves the parent encoded in `block`, `b2r(block)` retrieves the rank encoded in `block`, and `pr2b(parent,rank)` packs `parent` into the upper 56 bits and `rank` into the lower 8 bits and returns the resulting `block_t`.

```
static inline parent_t b2p(const block_t inblock) {
   return (parent_t)(inblock >> split_size);
};

static inline rank_t b2r(const block_t inblock) {
   return (rank_t)(inblock & rank_mask);
};

static inline block_t pr2b(const parent_t parent, const rank_t rank) {
   return (((block_t)parent) << split_size) | rank;
};
```

We restrict the record size for each element to 64 bits in order to avoid the performance penalty associated with non-native atomics. SOUFFLÉ is targeted at modern 64 bit Intel processors, which do not have performant 128 bit atomics, libatomic must be added as a dependency if 128 bit atomics are to be used. The graph in Figure 3.54 demonstrates the runtime disparity, for varying N as set by the micro-benchmark briefly described in Snippet 3.7. In Figure 3.54 both 32 and 64 bit atomics have the same runtime (shown overlaid), whereas 128 bit atomics are approximately 3 times slower for a million elements.

LISTING 3.7: Atomic performance benchmark

```
/* time the duration to store N 128 bit plain old data types*/
struct X128 { uint64_t a; uint64_t b; };
```

```
std::atomic<X128> x128;
for (size_t i = 0; i < N; ++i) x128.store(X128{i,i});

/* time the duration to store N 64 bit plain old data types*/
struct X64 { uint32_t a; uint32_t b; };
std::atomic<X64> x64;
for (size_t i = 0; i < N; ++i)
    x64.store(X64{static_cast<uint32_t>(i),static_cast<uint32_t>(i)});

/* time the duration to store N 32 bit plain old data types*/
struct X32 { uint16_t a; uint16_t b; };
std::atomic<X32> x32;
for (size_t i = 0; i < N; ++i)
    x32.store(X32{static_cast<uint16_t>(i),static_cast<uint16_t>(i)});
```

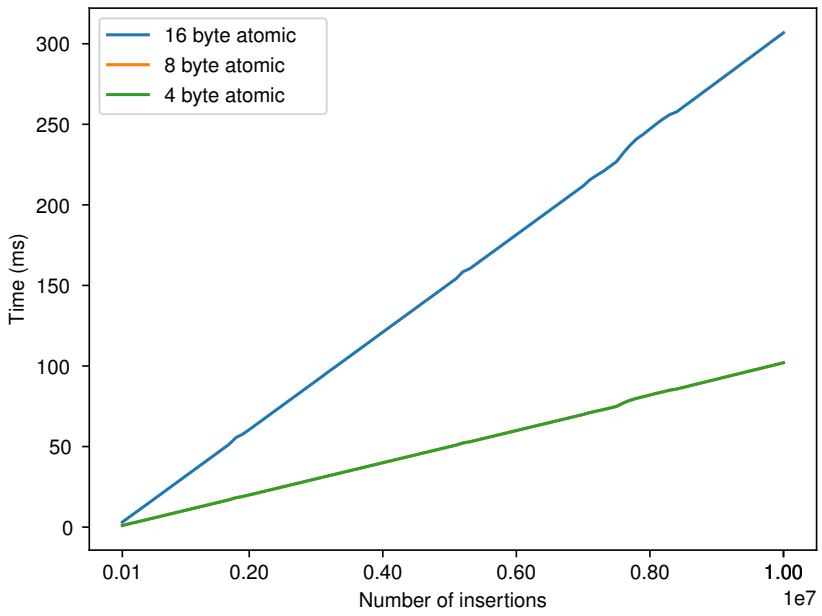


FIGURE 3.54: Store performance of various-sized atomic datatypes

By restricting ourselves to 64 bits to store both the rank and parent, we must assert that this is sufficient. From the result in Section 3.3.1, we see that $log_2 log_2(n)$ bits are necessary to store the rank. For 2^{64} elements, we require $log_2 log_2(2^{64}) = 6$ bits, although we round this up to 8 bits, to avoid necessitating sub-byte masking operations during every use of the `rank` field. Storing it in a byte is the logical

minimum required, natively supported as a single data-type in C++ as `uint8_t`; leaving 56 bits to store the `parent` field. Practical limits will be hit far before reaching 2^{56} elements in any data-structure, especially so in SOUFFLÉ - currently not all data-structures fully support 64bit data-types, so 56 bits is sufficient.

3.3.2.2 C++

C++11 formalised a standard memory-model, which we used as necessary to emulate the PRAM model as used by Anderson's disjoint-set data-structure. The PRAM model assumes all writes are atomic, and that modifications to global memory is visible instantly, whilst in C++ memory may become set to an invalid state when two threads attempt to update the same value simultaneously.

To overcome this atomic types may be used, although these suffer a performance penalty. Atomic types cannot be moved or copied in the C++11 context, this prevents their use in any data-structure this - such as a `std::vector`, or a `std::map`, and as such, pointers are often used to enable their storage. In our data-structure, we do not need to worry about this, as PiggyList never copies (nor moves) data, it guarantees that an elements will always be in the same location over the entire lifetime of the PiggyList.

Atomic types guarantee that modifications to the underlying data will not be lost. For the following program, if `increment` is called by two threads concurrently, it is possible that the resulting value of `counter` does not equal the number of invocations. On the other hand, `aCounter` carries that guarantee.

```
1  size_t counter = 0;
2  std::atomic<size_t> aCounter{0};
3
4  void increment() {
5     counter += 1;  // equivalent to x = fetch(counter), counter = x + 1;
          Two operations.
6  }
7
8  void atomicIncrement() {
9     aCounter.fetch_add(1) // atomically increment
10 }
```

The memory ordering of atomics by default enforces sequential consistency, which dictates that for any marked atomic operations, concurrent operations performed on them is equivalent to some sequential ordering. We do not require the strong guarantee of sequential consistency - the strongest memory order specified by the C++11 standard (Boehm and Adve, 2008) - instead we can relax to *acquire* and *release* semantics. All these enforce are *happens before* and *happens after* ordering relationships for operations on the same data. `x86` architectures do not require special instructions for these acquire and release orderings, as the architecture is designed with a strong ordering in mind. (McKenney, 2005)

The `COMPARE-AND-SWAP` operations have an equivalent primitive in C++11 , as a member function for all atomic types. `x.compare_exchange_strong(T& a, T b` and `x.compare_exchange_weak(T& a, T b)` both atomically set `x` to `b` iff `x = a`, although the weak function is allowed to fail spuriously.

The function signatures for the equivalent C++ disjoint-set operations are necessary to carry type definitions. As follows are the function declarations for `FIND`, `UPDATE-ROOT`, `SAME-SET`, `UNION`, and `NEW-ELEMENT`. Note that `makeNode()` returns a packed value for the newly created singleton.

```
1 parent_t findElement(parent_t x);
2 bool updateRoot(const parent_t x,
3                 const rank_t oldrank,
4                 const parent_t y,
5                 const rank_t newrank);
6 bool sameSet(parent_t x, parent_t y);
7 void unionNodes(parent_t x, parent_t y);
8 block_t makeNode();
```

3.3.3 PiggyList

Named due to the expanding nature of the data-structure, this is a new implementation and design of a concurrent vector. As SOUFFLÉ deals with monotonically increasing data-structures (i.e. they are only growing) except for clearing all elements, this allowed for a simpler, and more efficient data-structure. Despite this, the data-structure can easily be modified to allow deletion, and also be lock-free. For simplicity, the SOUFFLÉ implementation does not use these extensions.

The requirements of the wait-free Union-Find implementation, is that on contention it retries, requiring repeated `find(`u`)` calls that in turn necessitate random access to the storage container (i.e. access an element at index i), in order to perform path compression. A `std::vector` is ill-suited to this task, as concurrent writes or element creation that require expansion of the container capacity are not thread-safe. A linked-list never changes the location of elements and as such can accommodate concurrent writes and element creation; although the data-structure has poor random-lookup performance, but this can be abated by increasing the number of elements stored in each node of the linked list. By storing say 10000, elements within each node, the number of traversals required to be performed is cut down by a factor of 10000. Unfortunately, this is insufficient and still proves to be a bottleneck - using a larger number again helps, although at a significant memory overhead for smaller lists. Figure 3.55 details what we will call Blocklist, with chunk size k, also showing indexes $0, \ldots, k+2$ containing elements.

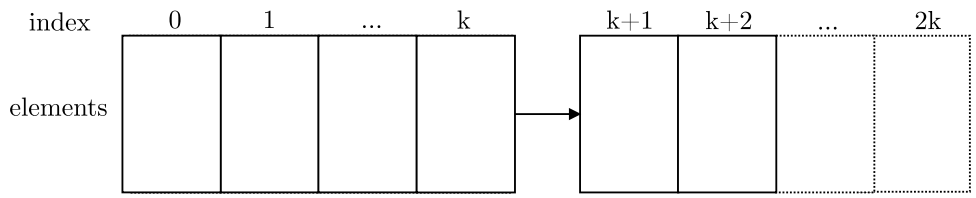


FIGURE 3.55: Basic chunked-linked-list that has an access bottleneck

To address this shortcoming, the initial container size can be small (given as r), and future nodes in the linked list use a size that is double the previous node's size (i.e. $2r$, then $4r$, and so on), to which elements can be inserted into. Starting with a container size of 1, the series of sizes per resize is $1, 3, 7, 15, ..., 2^n - 1$. Clearly, in a 64 bit system, no more than 64 nodes in our linked list will ever be required, which allows a 64 element long lookup-table, such that iteration is no-longer necessary.

Notably, this requires logarithms to identify which node a given index is, and would at first glance require $\mathcal{O}(logn)$ time. Fortunately, the integer logarithm base 2 can be done in a constant number of CPU cycles through the help of the gcc intrinsic `__builtin_clzll` (which counts the leading number of bits, subtract this from 64 (for 8 byte unsigned integers) to get the index in the lookup-table), or if not-compiling on gcc, the `x86_64` instruction `BSR` performs the same operation.

For sections of SOUFFLÉ that are expected to always store a larger series of elements, a bigger initial allocated block size can be used, as is in SOUFFLÉ , the default is an initial blocksize of 65535.

As previously mentioned, PiggyList does not move or copy any data. For the locking version, described in Section 3.3.3.1, it is not required to keep around the old versions so as to allow copying over to the

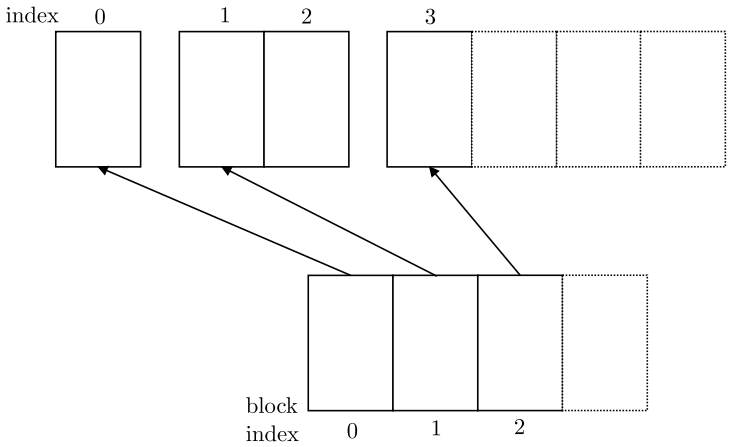


FIGURE 3.56: PiggyList with initial block size of 1; four elements have been created

new, larger container - strictly by design. This allows less memory overhead as compared to say, a `std::vector`.

3.3.3.1 Implementation & Interface

The implementation of the basic PiggyList requires the use of locks. Note that the value modifications are all performed atomically.

Algorithm 18 Appending an element to PiggyList

```
1:  procedure APPEND(element)
2:      index ← piggylist.size.fetch_add(1)
3:      if piggylist.container_size < index + 1 then        ▷ If this element's location doesn't exist
4:          piggylist.mutex.lock()
5:          while piggylist.container_size < index + 1 do        ▷ Double-checked lock
6:              ▷ Allocate the next block into the lookup-table
7:              piggylist.blocks[piggylist.num_containers] = new block(2^piggylist.num_containers)
8:              ▷ The next allocated block will be twice as big
9:              piggylist.container_size += 2^piggylist.num_containers
10:             piggylist.num_containers++
11:         end while
12:         piggylist.mutex.unlock()
13:     end if
14:     ▷ Set the element at its allocated index
15:     piggylist.blocks[⌊log2 index + 1⌋][index − (2^⌊log2 index+1⌋ − 1)] = element
16: end procedure
```

Algorithm 19 Retrieve an element in a PiggyList

```
1: procedure GET(index)
2:     return piggylist.blocks[⌊log2 index + 1⌋][index − (2^⌊log2 index+1⌋ − 1)]
3: end procedure
```

As PiggyList has the concept of indexes, iteration is trivial. An iterator can be constructed with its beginning at index 0, and terminating after index $size - 1$. A reverse iterator can be similarly constructed, with its initial index at $size - 1$, terminating after reaching index 0.

3.3.3.2 Additional Features

Element deletion. As previously mentioned, PiggyList can be implemented to accommodate deletion. In order to do so, a thread-safe linked list is also required, in order to store "deleted" elements' indexes (denoted in the following code as *deleted_nodes*). On element creation, if the linked-list is non-empty the current thread will attempt to pop from the linked-list, and if successful, the element will be inserted at the retrieved index, otherwise the element will be stored in the next available index (expanding if necessary).

Algorithm 20 Creating an element in a deletion enabled PiggyList

```
1: procedure APPEND(element)
2:     index ← piggylist.deleted_nodes.pop()
3:     if index then                                  ▷ Check if there is a deleted element to re-use
4:         ▷ Set the element at its allocated index
5:         piggylist.blocks[⌊log2 index + 1⌋][index − (2^⌊log2 index+1⌋ − 1)] = element
6:     else
7:         index ← piggylist.size.fetch_add(1)
8:         if piggylist.container_size < index + 1 then      ▷ If this element's location doesn't exist
9:             piggylist.mutex.lock()
10:            while piggylist.container_size < index + 1 do          ▷ Double-checked lock
11:                ▷ Allocate the next block into the lookup-table
12:                piggylist.blocks[piggylist.num_containers] = new block(2^piggylist.num_containers)
13:                ▷ The next allocated block will be twice as big
14:                piggylist.container_size += 2^piggylist.num_containers
15:                piggylist.num_containers++
16:            end while
17:            piggylist.mutex.unlock()
18:        end if
19:        ▷ Set the element at its allocated index
20:        piggylist.blocks[⌊log2 index + 1⌋][index − (2^⌊log2 index+1⌋ − 1)] = element
21:    end if
22: end procedure
```

To delete an element, the element can be destroyed at that index, and then the index is pushed onto the linked-list.

Algorithm 21 Deleting an element

```
1: procedure REMOVE(index)
2:     cleanup element at index
3:     ▷ signal that this index can be re-used
4:     piggylist.deleted_nodes.push(index)
5: end procedure
```

The Figures 3.57, 3.58, 3.59, 3.60 show the resulting state during the sequence of deletions: `delete(3)`, `delete(0)`, `delete(4)`.

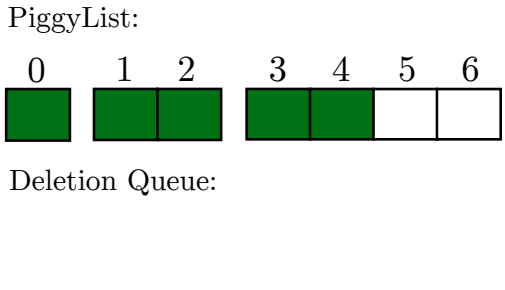


FIGURE 3.57: Starting PiggyList - occupied elements are labelled in green, unused in white

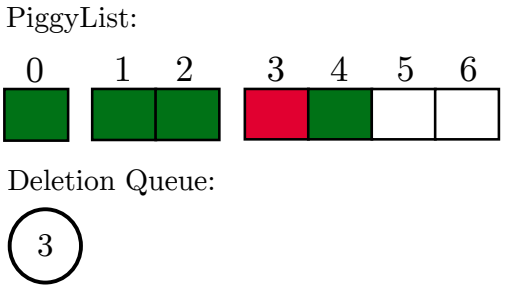


FIGURE 3.58: PiggyList after index 3 has been deleted. Red is used to mark deleted elements

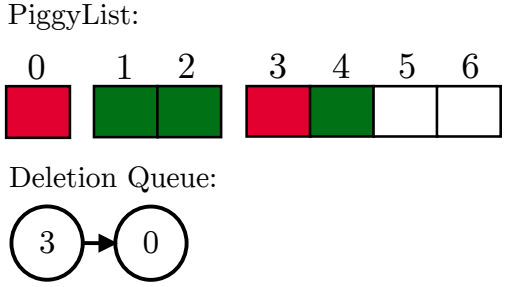


FIGURE 3.59: PiggyList after index 0 has been deleted

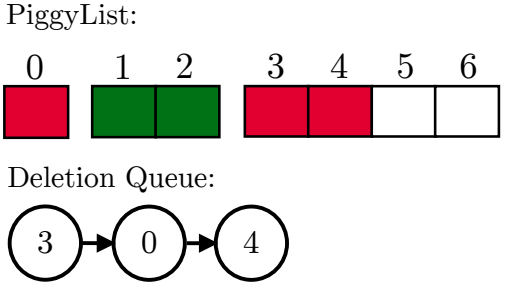


FIGURE 3.60: PiggyList after index 4 has been deleted

The next append operation will write into the first dequeued element from the front of the queue, in this case writing into index 3.

In order to reclaim space, the following non-thread-safe procedure may be used - this runs in $\mathcal{O}(m+n)$ time, where m is the number of deleted elements, and n is the number of total elements in the data-structure. If space reclaimation is not performed prior to iteration, it is necessary to insert tombstone or markers for deleted elements, such that they can be skipped when reached.

Algorithm 22 Reclaiming space

```
1:  procedure SHRINK(piggylist)
2:      final ← index of the last element in the piggylist
3:      for each e ∈ deleted queue do
4:          ▷ Only move elements towards the start of the list
5:          if e < final then
6:              move element at final into the index of e
7:              scan final back to point to the now-last element
8:          end if
9:      end for
10:     remove blocks without elements
11: end procedure
```

As follows is a pictorial representation of the trimming procedure, for the initial state as seen in Figure 3.61 which has elements at indices 1,4,5 already deleted.

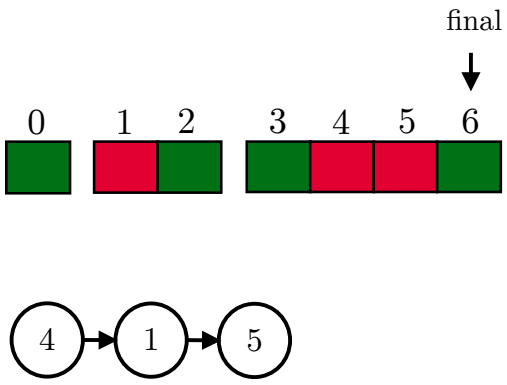


FIGURE 3.61: Starting PiggyList - the *final* used index is 6

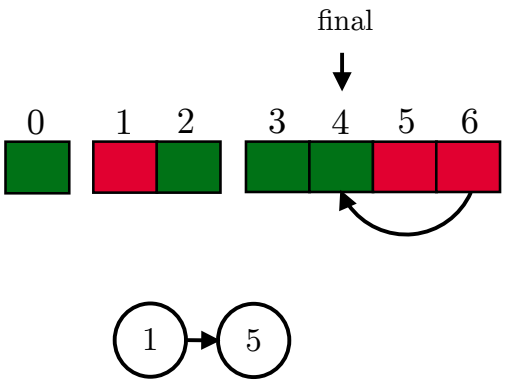


FIGURE 3.62: The element at index 6 is moved to index 4, final advances to the last element (4)

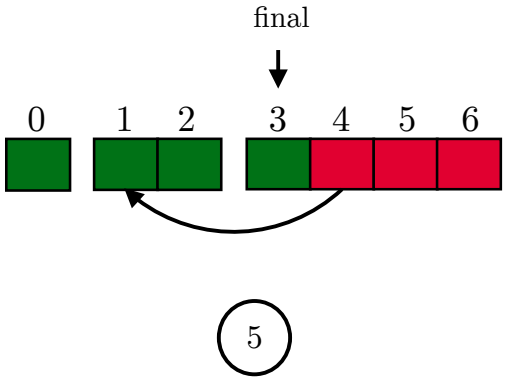


FIGURE 3.63: The element at index 4 is moved to index 1, final advances to index 3

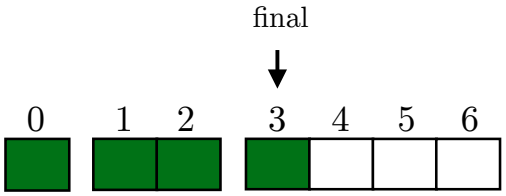


FIGURE 3.64: Index 5 is after the final index, so no elements are moved. We terminate as there is no more elements in the deletion queue

Wait-free. For a data-structure to carry the wait-free guarantee, each thread operating on the data-structure must be able to finish in a finite number of steps, independent on the other threads' operations. PiggyList can carry this guarantee, albeit with concessions in its worst-case space complexity. This can be abated by an extension, although this requires tuning to reduce contention as much as possible.

In order to be wait-free, the spin-locks that are used in container resizing must be replaced. When the requested index is larger than the current container size, a compare-and-swap operation is performed on the index of the lookup-table where the new segment would be placed. As follows is the pseudocode of the `makeNode()` function.

Unfortunately, when many threads attempt to expand the container size at the same time, many container-blocks will be allocated at the same time, which require deletion for the resulting failed threads. Worst-case space complexity in this scenario is now $\mathcal{O}(nt)$, with t denoting the number of threads. Empirical data of a small benchmark indicates this is not a rare occurrence, appearing to have a memory overhead of approximately that of the worst case nt, as demonstrated in Figure 3.65. It is important to keep in mind that instrumenting this memory usage undoubtedly increased the likelihood that a thread allocates memory whilst another is currently in between allocating memory and attempting to mark it as authoritative. A more accurate result may potentially be obtained by emulation, although this also carries complications.

Algorithm 23 Creating an element in a wait-free PiggyList

```
1:  procedure APPEND(piggylist, element)
2:      index ← piggylist.size.fetch_add(1)
3:      block_index ← ⌊log2(index+1)⌋
4:      if piggylist.blocks[block_index] is null then                ▷ Not allocated
5:          new_block ← new block(2^block_index)
6:          ▷ Try set allocated block if not already set
7:          if COMPARE-AND-SWAP(piggylist.blocks[block_index], null, new_block) = failure then
8:              clean up new_block
9:          end if
10:     end if
11:     ▷ Set the element at its allocated index
12:     piggylist.blocks[block_index][index − (2^block_index − 1)] = element
13: end procedure
```

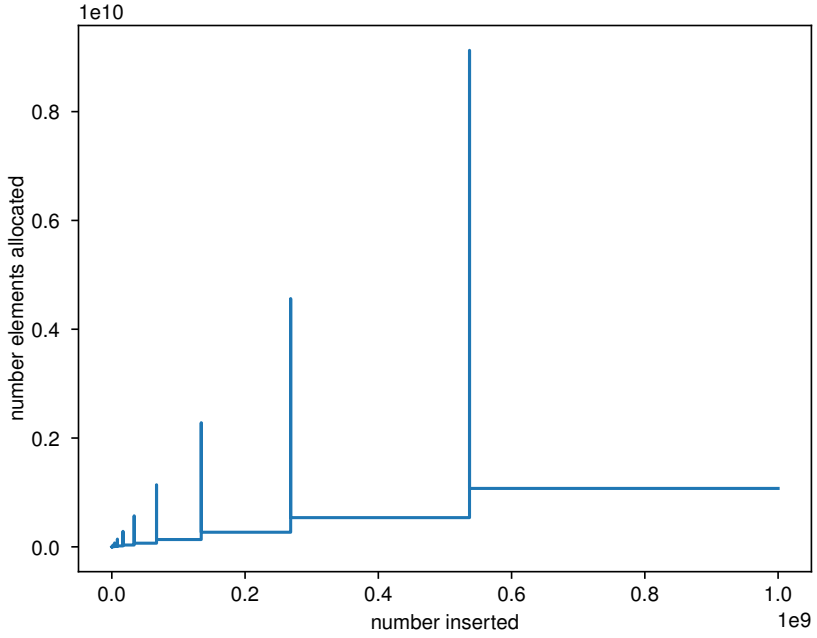


FIGURE 3.65: Memory consumption for a multithreaded insertion benchmark. The number of threads in this test is 16.

In order to reduce the chances of this occurring, a weighted dice-throw is conducted per element-creation that approaches 100% probability as the current container fills up. On a successful roll, the next container size is allocated, still using the lock-free resizing algorithm. Note that the probability is weighted such that it is not possible to allocate a new container that is not the current storing container's successor - i.e. if the current container is being added to, and a thread successfully rolls to create another container, although the next one has already been reserved, that thread will not create it.

Algorithm 24 Creating an element in a wait-free PiggyList

```
1: procedure APPEND(piggylist, element)
2:     index ← piggylist.size.fetch_add(1)
3:     block_index ← ⌊log2(index+1)⌋
4:     if piggylist.blocks[block_index + 1] is null then                ▷ Next block free
5:         remaining ← number of elements not occupied in this block
6:         if ROLL-DICE(remaining, 2^block_index, t) then
7:             new_block ← new block(2^(block_index+1))
8:             ▷ Try set allocated block if not already set
9:             if COMPARE-AND-SWAP(piggylist.blocks[block_index+1], null, new_block) = failure
   then
10:                clean up new_block
11:            end if
12:        end if
13:    end if
14:    ▷ Set the element at its allocated index, we assume current block is allocated
15:    piggylist.blocks[block_index][index − (2^block_index − 1)] = element
16: end procedure
```

The follwing dice roll predicate `dice-roll` provides a linear probability curve (visualised in Figure 3.66), in that as the current block fills up, the probability that a thread's dice-roll will succeed also increases. We coerce a guarantee that the threads must allocate if only t spaces are left in the container - this ensures that all threads can assume that their current block_index will be writeable, as there is no code-execution path for each thread that will allow no next block to be allocated, assuming t is known ahead of time.

Algorithm 25 Dice roll predicate, if a thread should try to allocate a new block

```
1: procedure DICE-ROLL(remaining, blocksize, thread_count)
2:     ▷ This ensures a thread's index's block always exists
3:     if remaining ≤ thread_count then
4:         return true
5:     end if
6:     return RAND([0, 1]) ≥ remaining / blocksize
7: end procedure
```

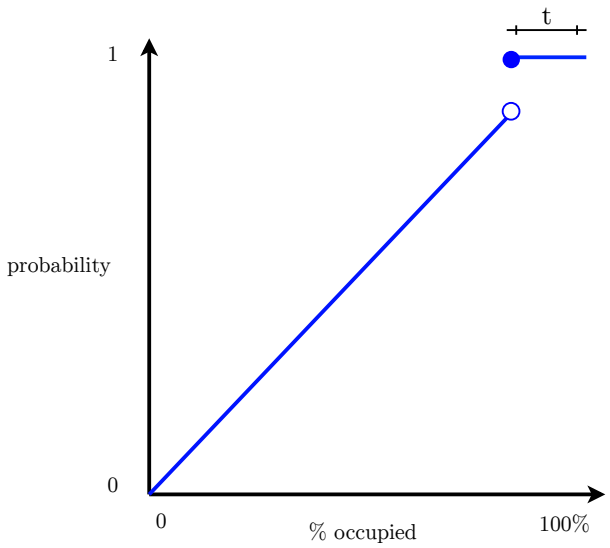


FIGURE 3.66: Probability graph of the linear dice roll function, the last t threads are guaranteed to succeed

For the current dice-roll function, the probability of two nodes creating a new container at the same time is dependent on many factors. For one, instructions may not necessarily take the same number of steps to complete, and even the same instruction may be slower or faster depending on the calling context, the scheduler, and other factors. If we try to assume that the threads will attempt to roll the dice at the same time (which would seem to be a likely scenario for duplicate memory consumption), even if only one thread succeeds, the thread may spend an arbitrary amount of time in the allocation block (i.e. within the body of the roll-dice conditional), and so other threads may have ample opportunities to retry the roll-dice functions on subsequent calls. Due to this, it is non-trivial to analyse complexity - even if it is attempted, the number of assumptions may make it unrepresentative of real-world performance. Empirical studies of probabilistic functions have proven sufficient in the literature of constructing concurrent data-structures.(Gibson and Gramoli, 2015) It is worth noting that the worst-case space complexity is still $\mathcal{O}(nt)$.

For this dice-roll predicate, the number of threads is known ahead of time as t. If for an application this is not known, a modified probability function can ignore t, and instead in the `append(e)` function, an additional check of the current block's existence is necessary. For ease of space complexity evaluation, the algorithm described in Algorithm 24 suffices.

An experiment into the space complexity of the probabilistic allocation approach using the linear probability function defined above is presented in Figure 3.67. Spurious allocations of 2 or 3x are

observed - a marked improvement over the ≈16x overhead of the non-probabilistic wait-free approach. As we see in Section 3.3.4, this overhead is similar to that of a typical `std::vector`.

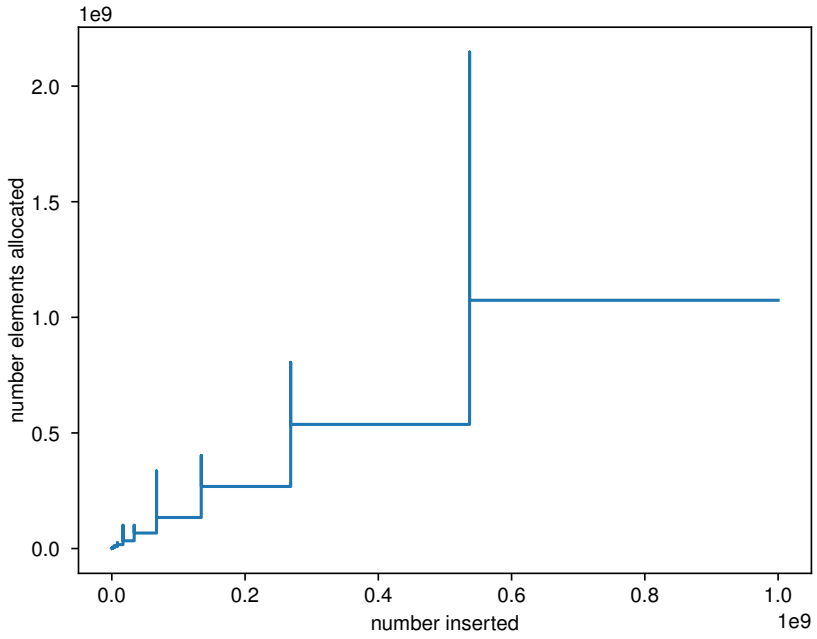


FIGURE 3.67: Space consumption of a uniform probability Wait-free PiggyList

3.3.4 Disjoint Set and PiggyList Benchmarks

In order to measure the difference in utility of the different data-structures, a variety of scenarios will be performed on the disjoint-set each using the different underlying data-structures. We only test the locking versions without deletion, as the extensions are currently not required as part of the overall data-structure.

We measure the performance of the disjoint-set on several operations:

- Read heavy, minimal write ($N-1$ consumers, 1 producer)
- Equal reading/writing ($N/2$ consumers, $N/2$ producers)
- Write heavy, minimal read ($N-1$ producers, 1 consumer)

The following benchmarks were performed on a 8 Threaded machine with a Intel(R) Core(TM) i7-7700K CPU @ 4.20GHz CPU.

Each thread operates on the same number of elements, disjoint from each other. In Snippet 3.8, the consumer performs read-only tests whether elements are within the same disjoint set. This is read-only

in the sense that the no path compression is performed, so no fields of the shared container are written to. Snippet 3.9 shows the equivalence producer thread - it simply unions elements to be in the same set. We union such that a large number of elements will be touched and have their fields updated. For the consumer/producer tests, we run all of these over 8 threads.

LISTING 3.8: Consumer Pseudocode

```
void consume(size_t operations) {
    for (size_t i = 1; i <
        operations; ++i) {
        disjointSet.readOnlySameSet(i,
            i-1);
    }
}
```

LISTING 3.9: Producer Pseudocode

```
void produce(size_t operations) {
    for (size_t i = 0; i <
        operations; ++i) {
        disjointSet.unionNodes(i,
            i/2);
    }
}
```

We graph the total timings of the operations for an 8 threaded run (N=8).

Read heavy operations appear to scale better for PiggyList compared to the BlockList, beating the runtime after around 2 million operations. The BlockList has block-size 10000 - if that is increased, we observed that if this increases, the runtime only slightly improves. Looking at equal consumers and producers shows that the BlockList scales much more poorly. The runtime of the `std::vector` is magnitudes slower than either the BlockList or PiggyList. This trend is continued throughout rest of these benchmarks.

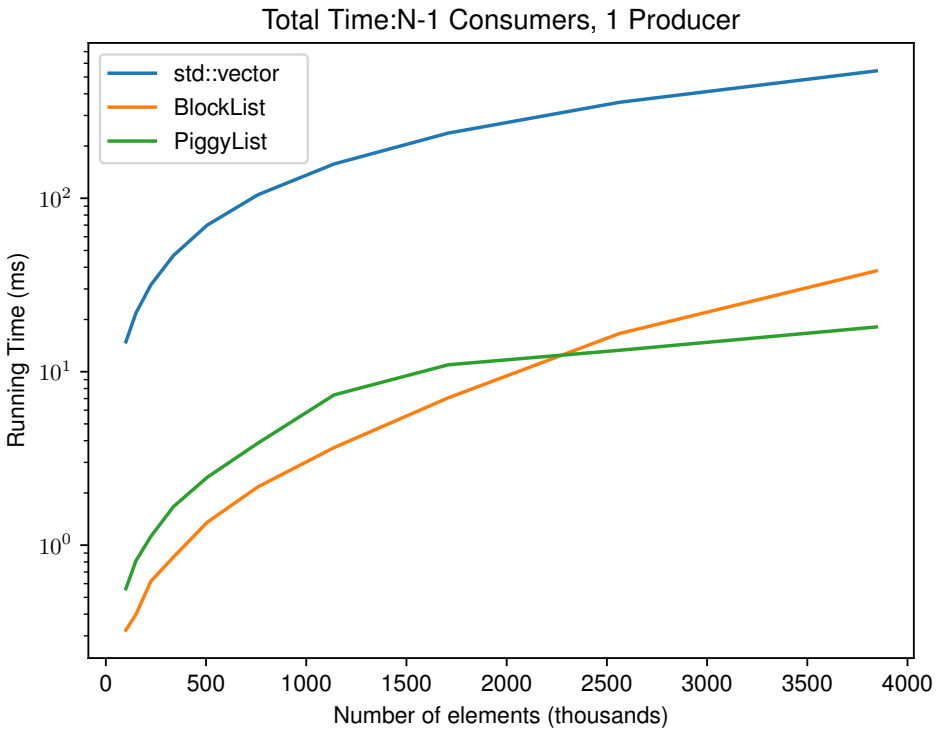


FIGURE 3.68: Runtime for read heavy concurrent operations

As the number of writers increases the performance of the BlockList degrades drastically, while the performance of PiggyList slightly decreases.

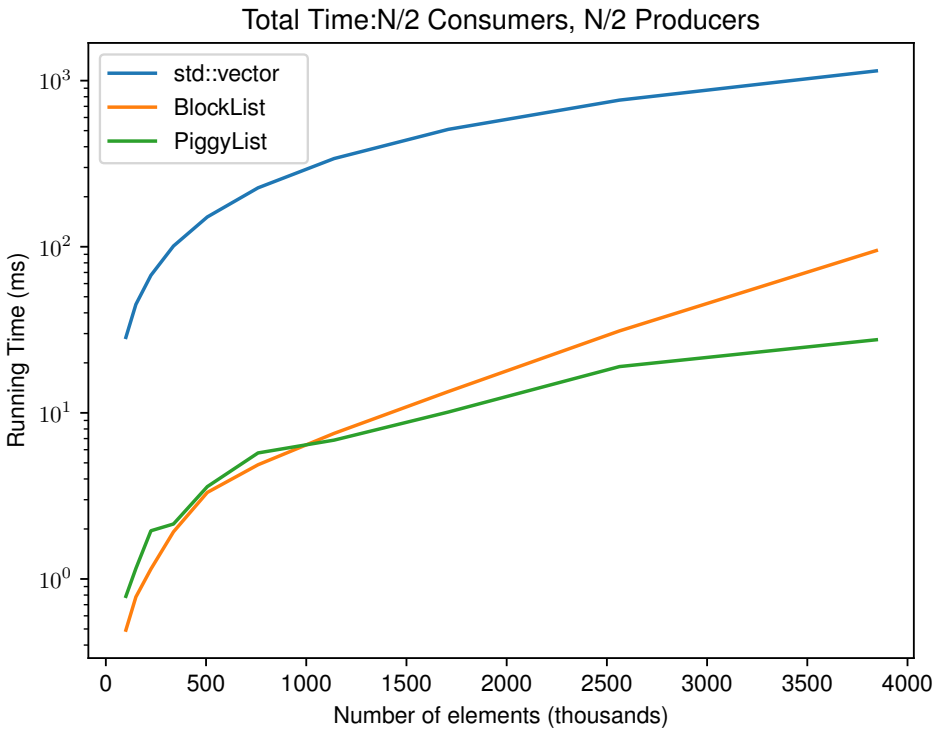


FIGURE 3.69: Runtime for equal read/write heavy concurrent operations

For a large number of writers, the BlockList appears to perform better than the PiggyList, however after around 1.3 million elements, the PiggyList supercedes.

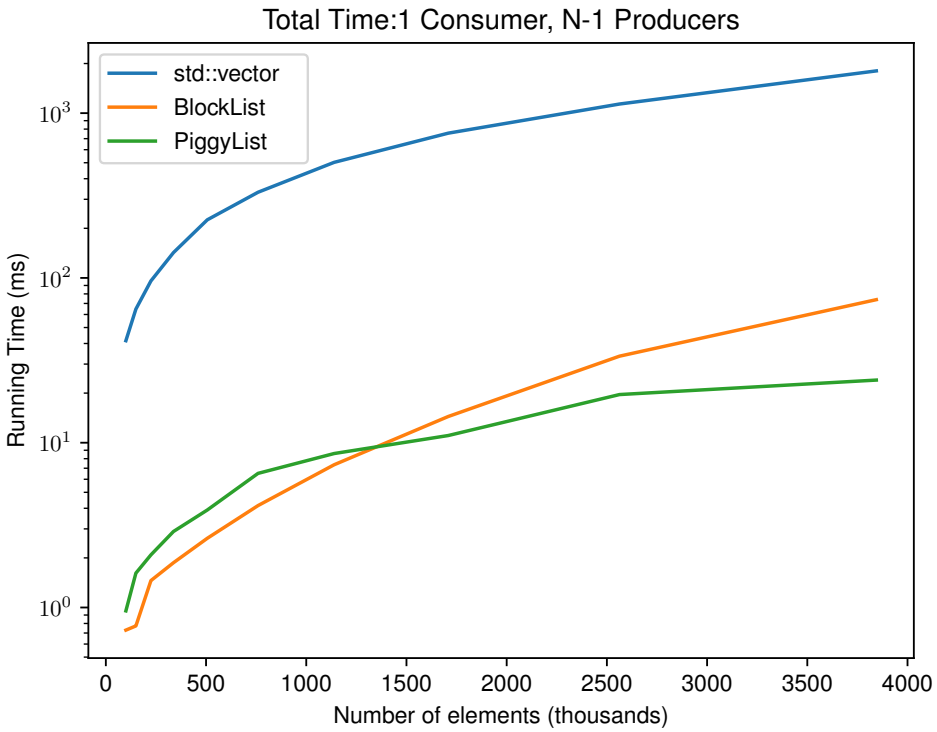


FIGURE 3.70: Runtime for write heavy concurrent operations

It is clear that the `std::vector` is vastly outclassed by the less-locking data-structures. In the following graphs we breakdown the mean runtime for each producer and consumer, highlighting the consequential interaction between different loads.

For the single producer benchmark, we see that the total runtime of the producing thread dominates for PiggyList, whose performance only supersedes the equivalent BlockList producer after 2.3 million elements. For the `std::vector`, both the producer and consumer threads have similar durations.

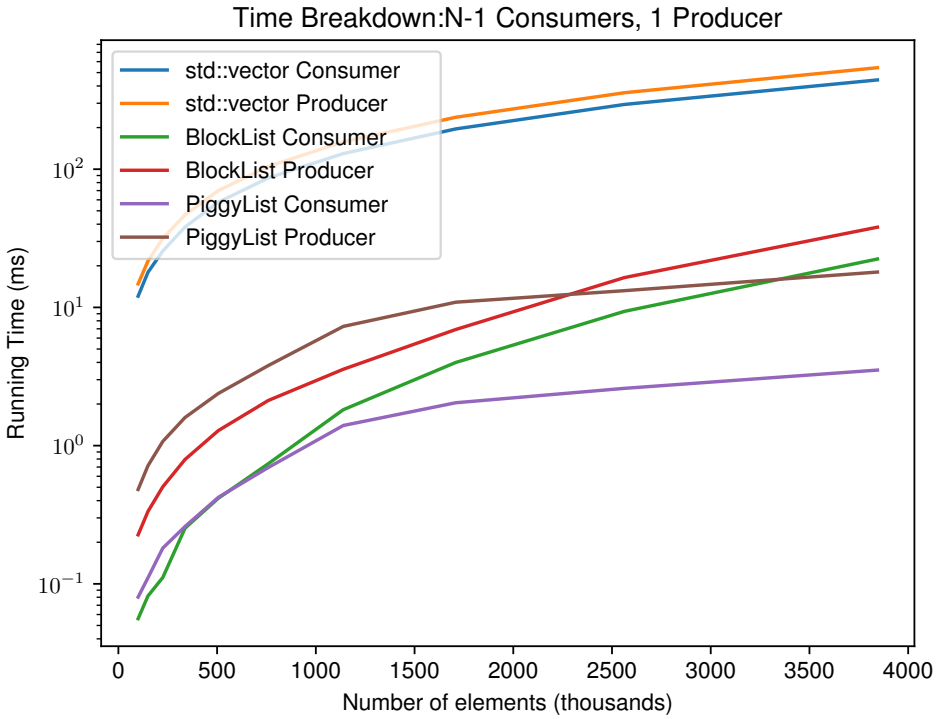


FIGURE 3.71: Mean Producer/Consumer runtime for read heavy concurrent operations

As the number of producers increase, the BlockList consumer threads seriously degrade in performance, consistently ranking slower than the producer thread. We see the similar reversing trend in PiggyList, where the producers rank faster than the consumer threads.

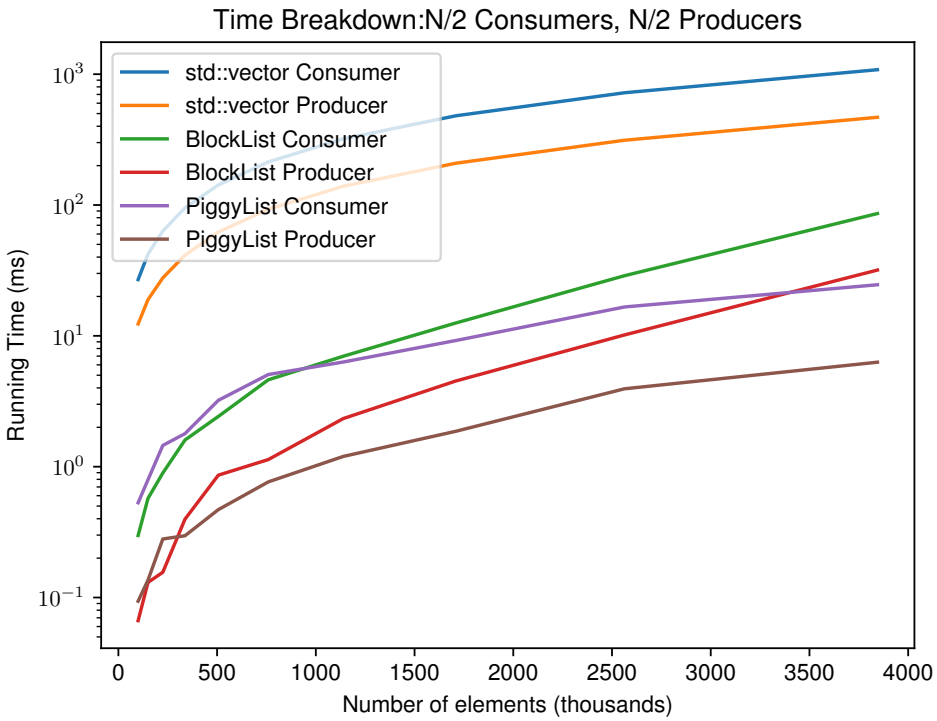


FIGURE 3.72: Mean Producer/Consumer runtime for equal read/write heavy concurrent operations

For the maximum number of producers, we see that the performance of the two operations are almost identical between all data-structures.

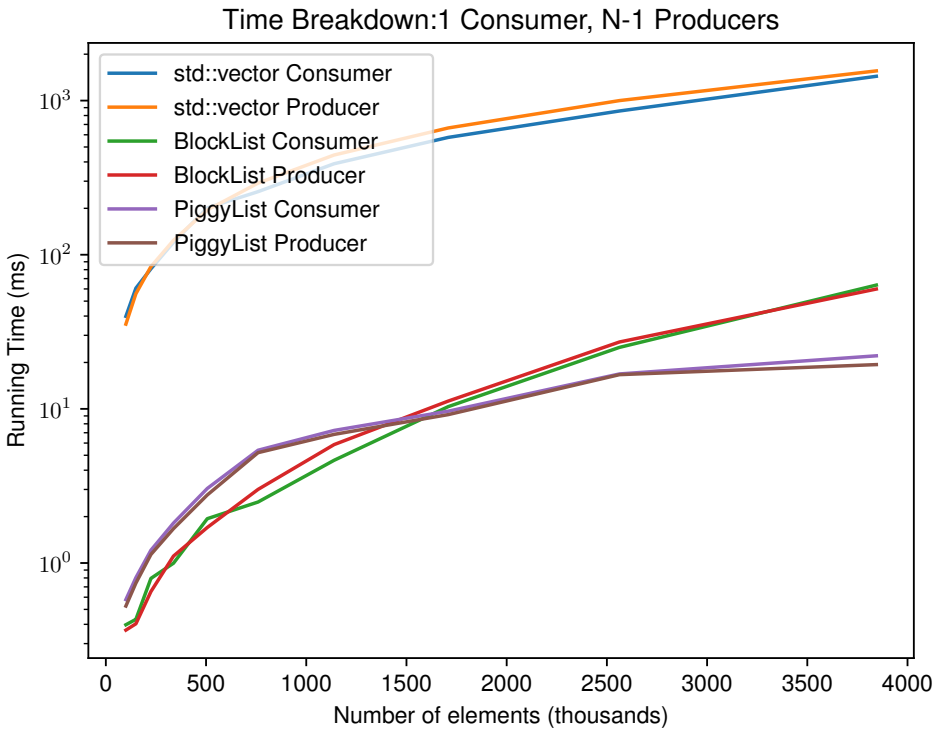


FIGURE 3.73: Mean Producer/Consumer runtime for write heavy concurrent operations

This demonstrates that the PiggyList has vastly better asymptotic complexity, however for small numbers of operations, the BlockList mostly outperforms. We see this to not be the case for threaded element creation - an often called operation during SOUFFLÉ operation.

In the following graphs we observe BlockList to consistently exhibit extremely poor performance for all thread counts - it is outperformed by `std::vector` protected by a lock. Interestingly, we don't see a linear speed-up for PiggyList, even for large number of elements. We suspect this is due to the highly contended atomic variable, which due to the x86 memory model, requires a relatively high level of synchronisation between threads - even if requested to operate with a relaxed memory model as would be possible with the PiggyList size variable.

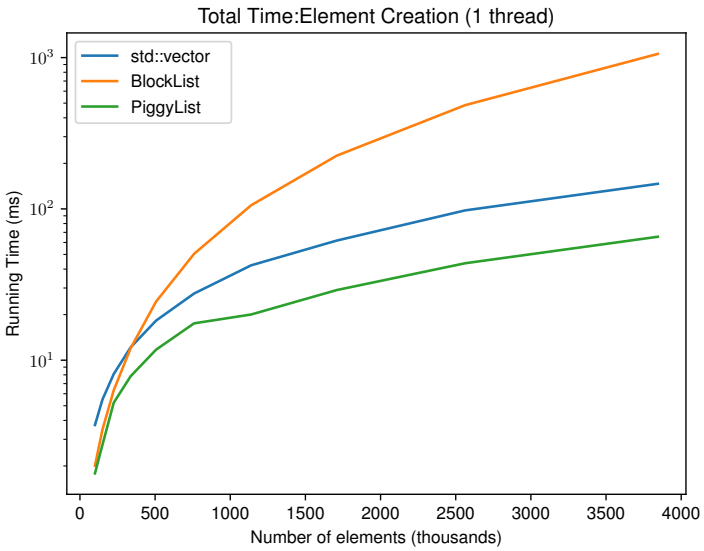


FIGURE 3.74: 1 Thread Insertion

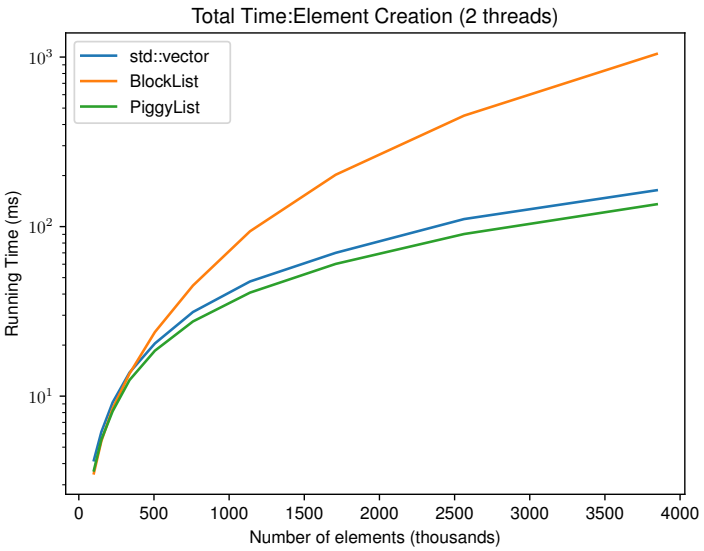


FIGURE 3.75: 2 Threaded Parallel Insertion

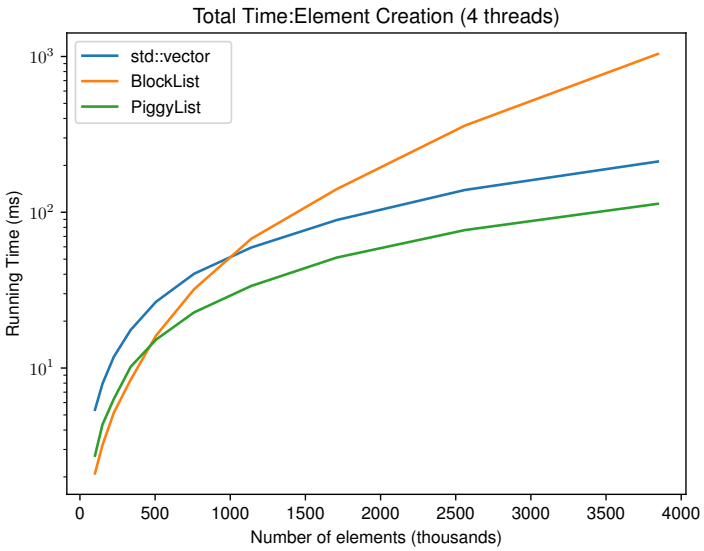


FIGURE 3.76: 4 Threaded Parallel Insertion

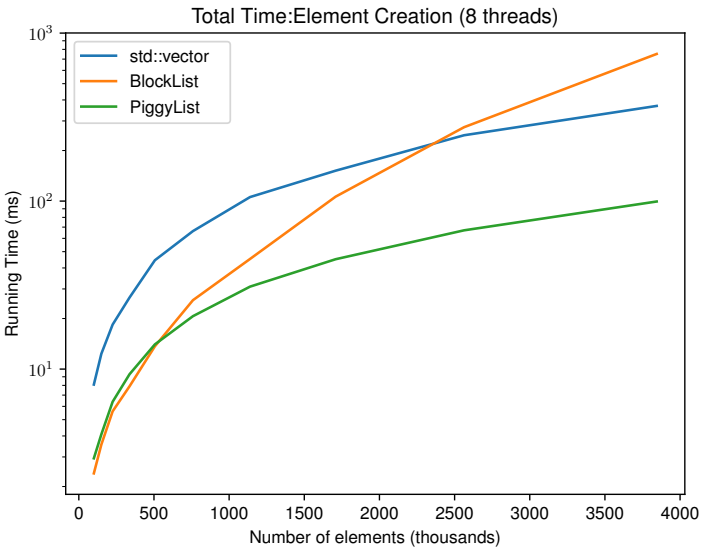


FIGURE 3.77: 8 Threaded Parallel Thread Insertion

The worst case memory consumption for the Locking PiggyList with deletion is the maximum number of elements stored at once - even if the elements are removed one-by-one, as there is no de-allocation of reserved blocks (unless shrinking is performed). As mentioned above, the wait-free PiggyList (with or

without deletion) may use at most $\mathcal{O}(nt)$ space, in the scenario that all t threads attempt to allocate more space, of which $t-1$ threads later delete.

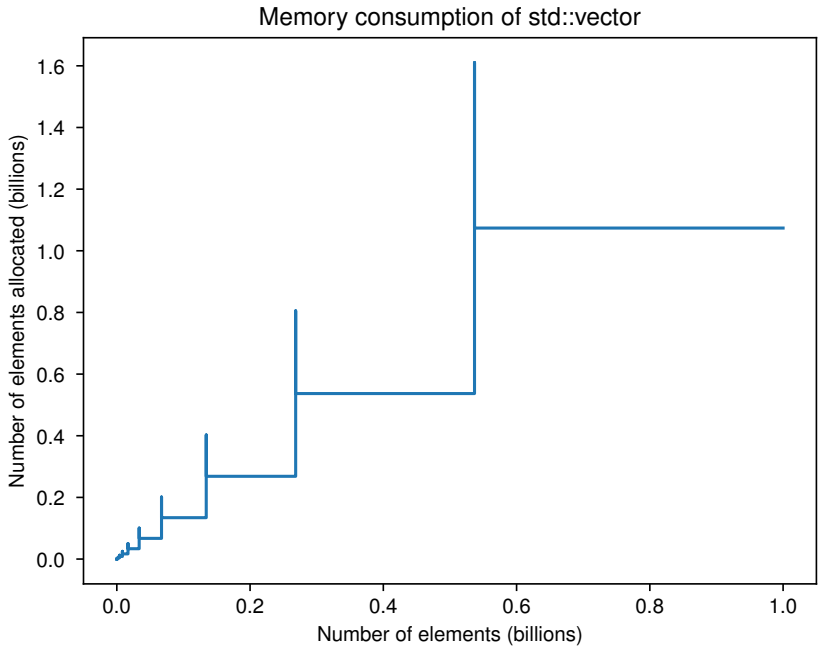


FIGURE 3.78: Memory consumption of a `std::vector` as elements are inserted

It is interesting to compare the performance of the probabilistic wait-free PiggyList to a standard `std::vector`. In Figure 3.78, we see similar memory overhead for the STL container, as compared to the probabilistic wait-free PiggyList in practice in Figure 3.67. The locking PiggyList's memory overhead is tight, with no 'spiked'-overhead when the container expands, shown in Figure 3.79. It is important to note however, that this spiking is only consistently always observed within `std::vector` for when non-trivially copyable objects are stored within the container. For trivially copyable types a `realloc` will be attempted, so that if possible, the container is resized in-place, and no overhead will be seen (in addition to not requiring copies of the objects stored). It is not simple to benchmark this, and it varies wildly depending on the available RAM on the computer, the operating system, and the address space variations induced by ASLR.

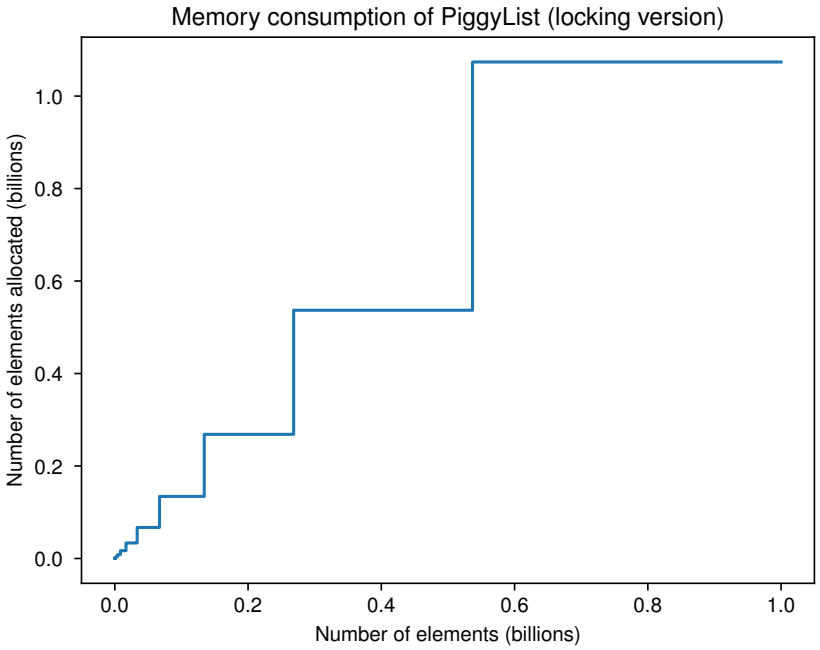


FIGURE 3.79: Memory consumption of a Locking PiggyList as elements are inserted

Single-threaded append performance: due to the lack of copying objects on resize, the PiggyList will sometimes outperform a contiguous `std::vector` depending on the size of the data stored. In the following benchmarks (Figures 3.80, 3.81, 3.82, 3.83) we measure the comparative performance of `std::vector::push_back` and a modified `PiggyList::append` optimised for single-threaded usage.

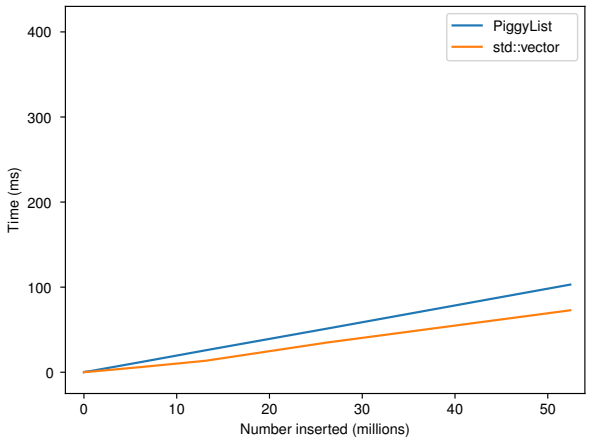


FIGURE 3.80: 8 bit (`uint8_t`) append performance

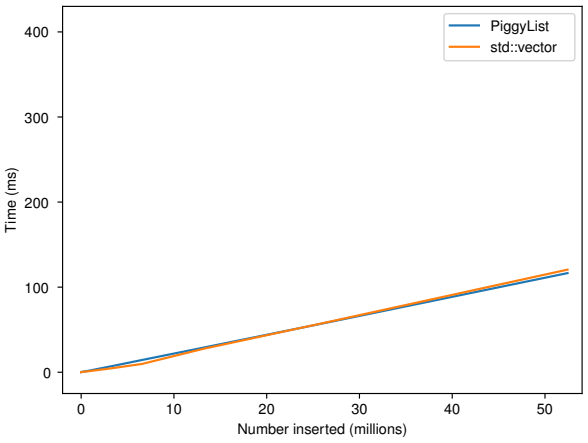


FIGURE 3.81: 16 bit (`uint16_t`) append performance

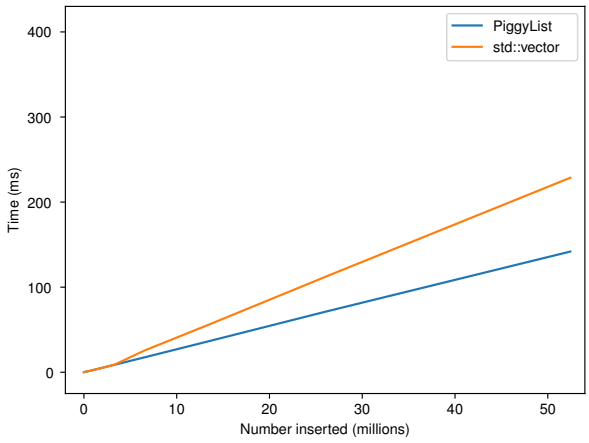


FIGURE 3.82: 32 bit (`uint32_t`) append performance

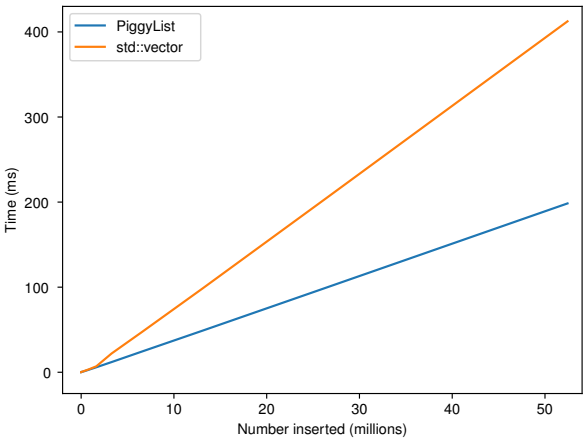


FIGURE 3.83: 64 bit (`uint64_t`) append performance

For 16 bit data-types and above, we observe an improvement in append performance. The benefit in 16 bits is marginal, for 32 bit a `std::vector` is just under 50% slower, and for 64 bit the `std::vector` is just over half the speed of the PiggyList.

Whilst this tests only primitives, PiggyList also demonstrates improved performance for storing objects that have costly move- or copy-constructors. PiggyList also has the benefit of being allowed to store non-copyable or non-moveable types, such as `std::atomic`. In order to store these in a STL container, a

pointer instead must be stored. This proves expensive as these would all require separate and independent allocation, in addition to the allocation of new elements as the container expands. This is only applicable to PiggyList implementations that do not perform node deletion, as supporting that requires movement or copying of stored objects.

CHAPTER 4

Experiments

In order to demonstrate the use cases for native equivalence relations in Datalog, a series of experiments have been set up. We compare the run-time of the implicit representation of equivalence relations to the explicit representation in a real-world benchmark - identifying users from the Bitcoin blockchain. For points-to analysis, we compare two different types of analyses, a subset based analysis and an equivalence analysis - where the latter had not been able to be implemented in SOUFFLÉ until now due the associated computational cost in a declarative programming language.

Datalog has only recently demonstrated productive examples of points-to analysis over billions of tuples, as early as 2016. (Scholz et al., 2016) This was also performed in the SOUFFLÉ suite, on a similar input domain as our points-to analysis performed in Section 4.2 on the OpenJDK. We demonstrate productive examples of analyses generating a *trillion* tuples in under four seconds.

4.1 Bitcoin User Groups

Bitcoin provides a pseudonymous way of digital transactions, wherein transfers of the Bitcoin currency are enabled through public transactions published on the blockchain. Figure 4.1 provides a simplified view of how each published block (occurring approximately every 10 minutes) contains a series of transactions, a hash of the previous block and additional data.

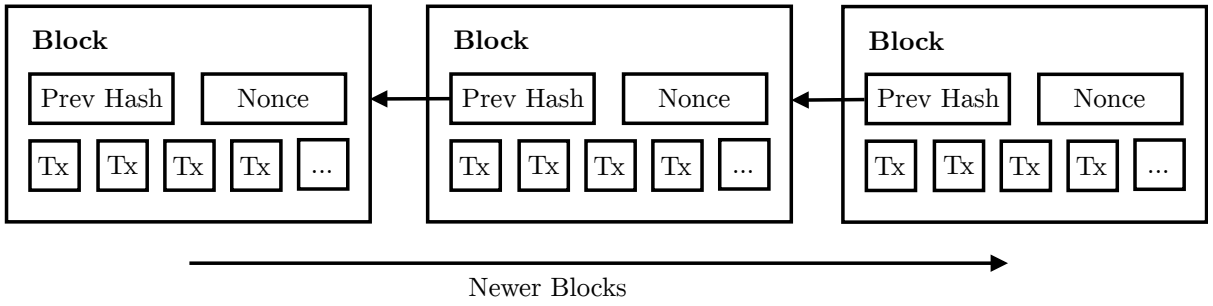


FIGURE 4.1: The blockchain structure (Modified from (Nakamoto, 2008))

For our purposes, we consider each transaction to contain a series of input identifiers which are the public keys corresponding to the private keys used to sign the transaction, a series of output identifiers which are the public keys of the receiving users and the quantity of Bitcoin to send them each, and finally a hash of the entire transaction. We picture a simplified transaction in Figure 4.2.

Addresses In:

1AHBN8YuQVqKLzZo4V2L5WCxewwr7xtDHS
1HvhZyxtJfSWxDJjcpM8THfQ3yB6GPxZX9

Addresses Out:

16ZdsGm3J9uTt5K7N8ojW8D66X7kMJCytS
1Lq5DVmok8p9opyy5QFBX2cM48xTELecaj

Transaction Hash

c1c50ac324f45d0ecc2d8b1708666ca43ea894ebaf9cb79aceba914867c029c5

FIGURE 4.2: A simplified Bitcoin transaction

The public keys have an associated private key, which must be used in order to ensure authenticity of that action via the act of *signing* - similar to how an ordinary cheque requires the signature of the sender in order to be valid. This private key must be secret, otherwise an adversary will be able to create transactions on behalf of the private key owner. A demonstration of the signing, and associated verification is pictured in Figure 4.3. The public key is considered public, that is, anyone is able to possess it without consequence - and in fact is required to if they wish to confirm the validity of the transaction. A new private-public key pair is typically generated per transaction, as this is considered good practice for a payee. (Reid and Harrigan, 2013) The word *wallet* is often used to describe addresses that have Bitcoin amounts associated with them, analogous to a wallet that contains cash. After the transaction is signed, it

is forwarded on to the Bitcoin network to be added into the next available block (provided it has provided a large enough transaction free & the transaction is valid).

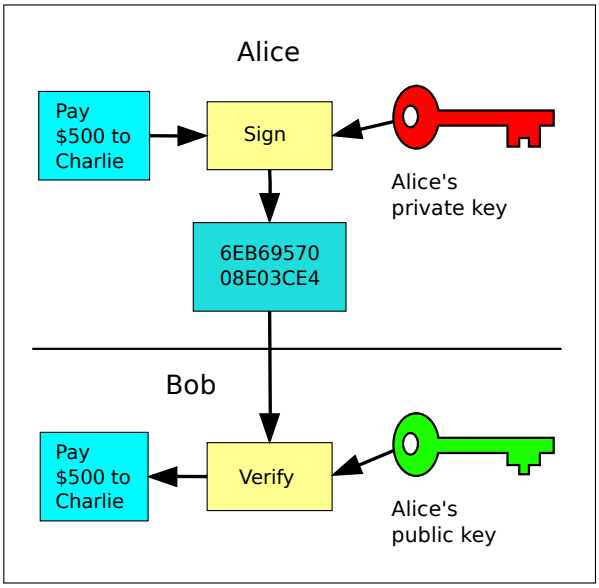


FIGURE 4.3: Signing a transaction with a private key (Modified from (GÃűthberg, 2006))

As the private key must be secret, we are able to consider all transactions associated with the corresponding public key to have been created by the same user. For any meaningful analysis over the user distribution, it is necessary to group public keys by the user that owns them. Reid and Harrigan proposed considering all public keys that are input to the same transaction to be controlled by a single person, as they require control of the associated private keys. (Reid and Harrigan, 2013) The website WalletExplorer (Janda, 2013) uses this heuristic in order to identify Bitcoin exchanges (sites that are used to trade fiat currencies with cryptocurrencies like Bitcoin). (jstolfi , https://www.reddit.com/user/jstolfi) As noted, this is inaccurate due to some Bitcoin services handling private keys and mix inputs together as part of their methodology.

Notably, this heuristic can be represented by an equivalence relation, as it is reflexive (public keys are owned by the same user as their own user), symmetric (likewise), and transitive (inputs across multiple transactions may be shared). We wish to demonstrate the efficiency of the implicit representation over the previous method of explicit representation of equivalence relations.

4.1.1 Input Dataset

As our dataset, we use a subset of all Bitcoin transactions from 2017 - there are over 200 million transaction/input pairs that year. We are only able to analyse a subset of the transactions due to the computational and space requirements of doing so. In Table 4.1, the left-hand column denotes how many pairs are loaded in as facts. This data was scraped using the high-performance BlockSci tool (Kalodner et al., 2017), which enables simple interaction with the large blockchain dataset, and provides a multi-threaded method to extract data.

Size	PubKeys	Transactions	Classes	Singletons	Largest Class	Mean Size	Same User Pairs
1000	971	935	906	854	5	1.07	1135
5000	4685	4117	3803	3303	11	1.23	7947
10000	9141	7516	6662	5726	42	1.37	26965
50000	40893	29881	24451	21929	1420	1.67	3764247
100000	76129	54555	43335	38185	2768	1.76	13351193
500000	343008	236291	171874	144299	9675	2.00	286292918
1000000	717098	481599	340467	279324	29344	2.11	1465193896
5000000	3442156	2737910	1776519	1469558	61384	1.94	12096911888
10000000	6631620	5461708	3411035	2834282	124007	1.94	49007650539

TABLE 4.1: Statistics of the input data set

Whilst large-scale Datalog analyses have been performed, these examples did not contain examples of transitivity which is what we believe to be the largest contributor of program execution time in our run of the explicit representation, as described earlier in Section 2.3.

4.1.2 Datalog Programs

To demonstrate the conciseness of implementing this experiment, Snippet 4.1 is the Datalog program used for the implicit equivalence relation. We mark the `same_user(user1, user2)` relation as an equivalence relation (`eqrel`) to induce the reflexive, symmetric, and transitive nature of this relationship between transactions. We make modifications to the generated C++ file in order to benchmark the individual components of the runtime: I/O, solving, and enumeration over all pairs. For the enumeration over all pairs, it was necessary to omit writing the output and instead increment an atomic integer, in order to focus entirely on the speed of iteration, thus removing the bottleneck of disk I/O. It was not necessary to speed up for input (perhaps via mapping the fact file into a RAM disk), as these facts from the extensional database (EDB) were loaded into a regular relation, identical between both implicit and

explicit equivalence relation programs - and thus do not interfere with the experiment as we discard these index timing from the total compared solving time.

Iteration over the final output pairs (in this case, the pairs of users that are classified as the same) is done in a single thread, as it typically is written to disk. We modify this to occur in parallel through augmenting the generated C++ file, to simulate the rule being fed into another rule. This allows us to compare the iteration speeds over the relation across implementations.

These experiments were each run 5 times each, and averaged out except for those exceeding computational time or crashing, as marked.

LISTING 4.1: Implicit Equivalence Relation Benchmark Program

```
1   .type TxId
2   .type PubKey
3   .decl transaction_input(tx : TxId, userIn : PubKey)
4   .input transaction_input()
5   // whether two users are the same
6   .decl same_user(user1 : PubKey, user2 : PubKey) eqrel
7   .output same_user()
8
9   same_user(u1, u2) :-
10     transaction_input(tx, u1),
11     transaction_input(tx, u2).
```

The comparative explicit implementation requires an additional two rules for symmetry and transitivity to form an equivalence relation - reflexivity is not required, as within the `same_user_explicit(u1, u2)` rule on line 9, there is no restriction that `u1` and `u2` must be different.

LISTING 4.2: Explicit Equivalence Relation Benchmark Program

```
1   .type TxId
2   .type PubKey
3   .decl transaction_input(tx : TxId, userIn : PubKey)
4   .input transaction_input()
5   // whether two users are the same
6   .decl same_user_explicit(user1 : PubKey, user2 : PubKey)
```

```
7    .output same_user()
8
9    same_user_explicit(u1, u2) :-
10      transaction_input(tx, u1),
11      transaction_input(tx, u2).
12   // symmetry
13   same_user_explicit(u1, u2) :- same_user_explicit(u2, u1).
14   // transitivity
15   same_user_explicit(u1, u3) :- same_user_explicit(u1, u2),
        same_user_explicit(u2, u3).
```

4.1.3 Results

This experiment also demonstrates the current cost of reading in facts, in Figure 4.4 we demonstrate the cost of loading in these input tuples from disk, as well as the associated memory of the program up until that point. We observe a rough proportional relationship between fact read in time and memory consumption of the loaded facts, consistent with the internal storage implementation of SOUFFLÉ . We use the GNU `time` program (version 1.7) to measure this, using the `-f "%e"` flag, which prints the peak RSS in kilobytes, (Eddelbuettel, 2000) and terminating the program after all the threads have been read from file.

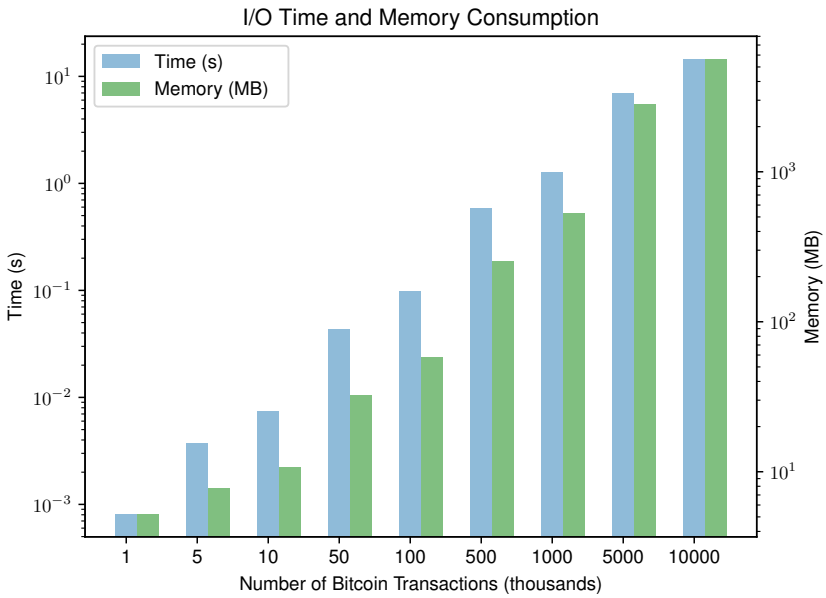


FIGURE 4.4: Memory and time consumption of the input symbol table

SOUFFLÉ transforms input terms from symbols to numbers within the RamDomain, that is, any term of a fact is mapped to an integer, similar to the densification step in our data-structure. Whilst this does mean that the arguments inserted into the equivalence relation data-structure are assigned ordinal numbers twice, this is necessary as if other facts or symbols are read in from file (or in fact generated via concatentation and aggregations in Datalog) before, the arguments inserted into the equivalence relation are no longer the sequential numbers starting from 0. On top of this, numeric types (`.type number`) do not have mapping applied to them, so inserting these into the equivalence relation requires densification in order to ensure a low footprint.

This mapping from symbol to numbers is not well optimised - it uses a `std::unordered_map` on a single thread, with all files loaded in sequentially. A future improvement to SOUFFLÉ may include optimising this to perform I/O in parallel when possible. Internal code profiling showed that the hash-map was the bottleneck compared to disk read time, and thus could be replaced with a concurrent equivalent, wherein a busy-queue can insert into, where the queue is populated from the fact files.

Figure 4.5 (log scale) demonstrates the *solving* time for the `same_user` predicate, which excludes reading facts from file, and iterating over the final derived facts.

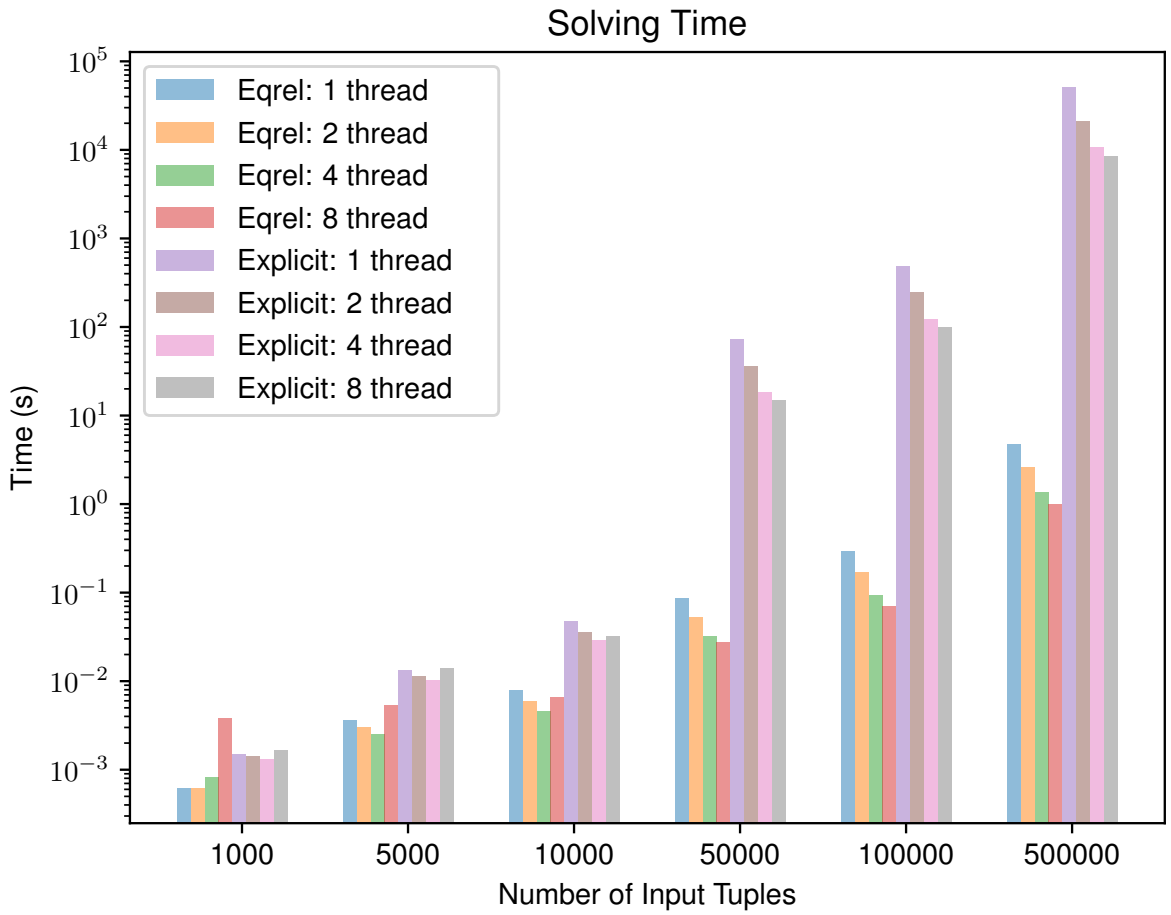


FIGURE 4.5: Solving time for the `same_user*` predicate for the Bitcoin data set

	Eqrel				Explicit			
Input Size	Threads							
	1	2	4	8	1	2	4	8
1000	0.00062	0.00062	0.00081	0.0038	0.0015	0.0014	0.0013	0.0017
5000	0.0036	0.0030	0.0025	0.0054	0.013	0.011	0.01	0.014
10000	0.0078	0.0060	0.0046	0.0066	0.048	0.036	0.029	0.032
50000	0.087	0.053	0.032	0.027	72	36	18	15
100000	0.29	0.17	0.093	0.070	490	240	120	100
500000	4.7	2.6	1.4	0.99	51000*	21000*	11000*	8600*
1000000	10	5.6	2.9	2.1	-	-	-	-
5000000	46	25	13	9.4	-	-	-	-
10000000	93	51	26	19	-	-	-	-

TABLE 4.2: Solving Time (seconds, 2 s.f.), * indicates the experiments were only ran once

The data indicates the implicit representation grows linearly with the size of the input domain, except for a 16x jump in solving time between 100000 and 500000 input tuples. We are currently unaware of the cause for this, which indicates additional internal profiling may necessary to investigate further, although the discrepancy is only minor ($\approx$ 3x). The relationship between the input domain size and the solving time for the single threaded implicit implementation is demonstrated in Figure 4.6.

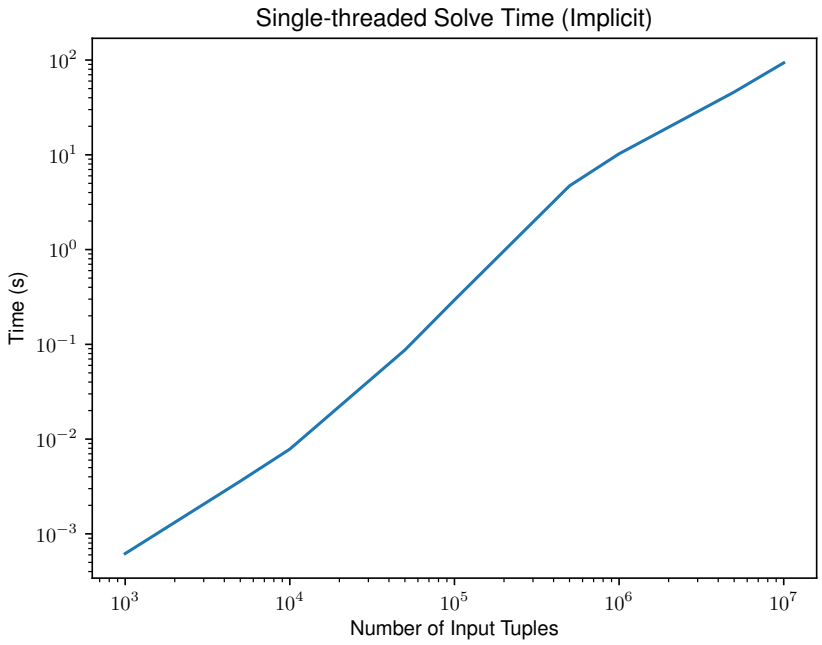


FIGURE 4.6: Solving time of the implicit program vs. input tuples

Whilst we were not able to extend the experiment for larger tuples in the program using the explicit representation, we believe the relationship is linear to the number of *output* tuples, which grows relatively quadratic to the input tuples. Figure 4.7 demonstrates the relationship between the solving execution time and the number of output tuples for the explicit representation.

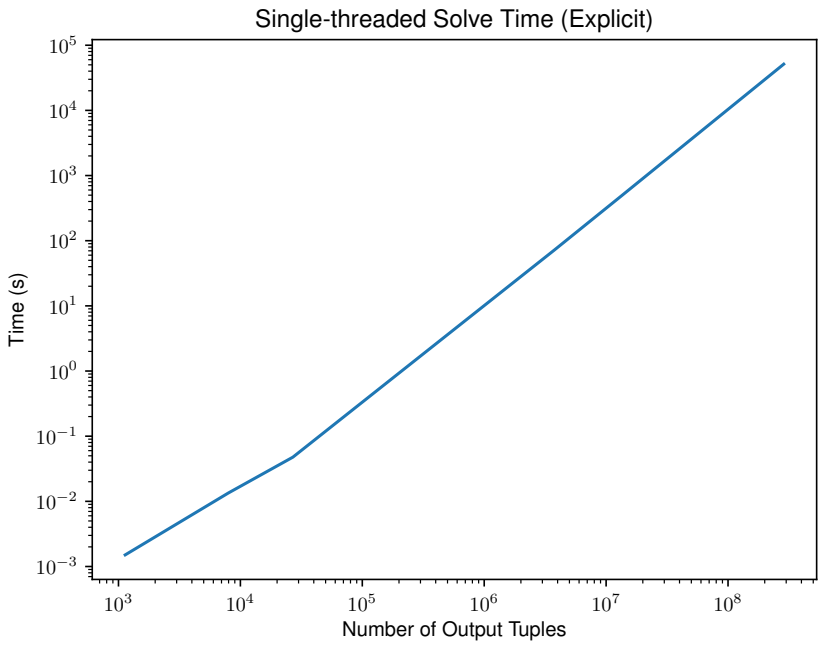


FIGURE 4.7: Solving time of the explicit program vs. output tuples

In addition to the poor running time, the number of generated tuples for the explicit version is inconsistent with both the implicit implementation and a custom imperative implementation of the program. This occurs for all numbers of threads, indicating that it is not a race condition, but instead an issue with handling large programs. Interestingly the single threaded explicit program was the *most* inaccurate, generating 319687058 output pairs compared to the correct amount of 286292918, a 12% over-approximation. The 2, 4, and 8 threaded programs each generated incorrect numbers of output tuples, albeit within a 1% error margin.

We also observed the eqrel (implicit) programs for 10 million input tuples crashing occasionally, either due to exceptions or segmentation faults. 1 out of 5 of the single threaded runs segfaulted, no two-threaded runs crashed, 1 of the 5 four-threaded runs caused an exception, and 2 of 5 runs of the eight-threaded run segfaulted. We are currently unaware of the cause of this behaviour.

Iteration time over the pairs indicates an overhead for the smaller sized equivalence relations in the implicit representation, gradually matching the performance as the number of output tuples increases. This overhead is most apparent for operation over 1000 to 10000 input tuples inclusive, which we attribute to the constant time overhead of managing a hash-map and lists in generating the equivalence cache in the implicit representation program. Figure 4.8 demonstrates the timings against the number of input tuples.

Note that the iteration is for an incorrect number of tuples for the 500000 input tuple explicit program, which potentially provides a 12% slowdown for the explicit single-threaded program.

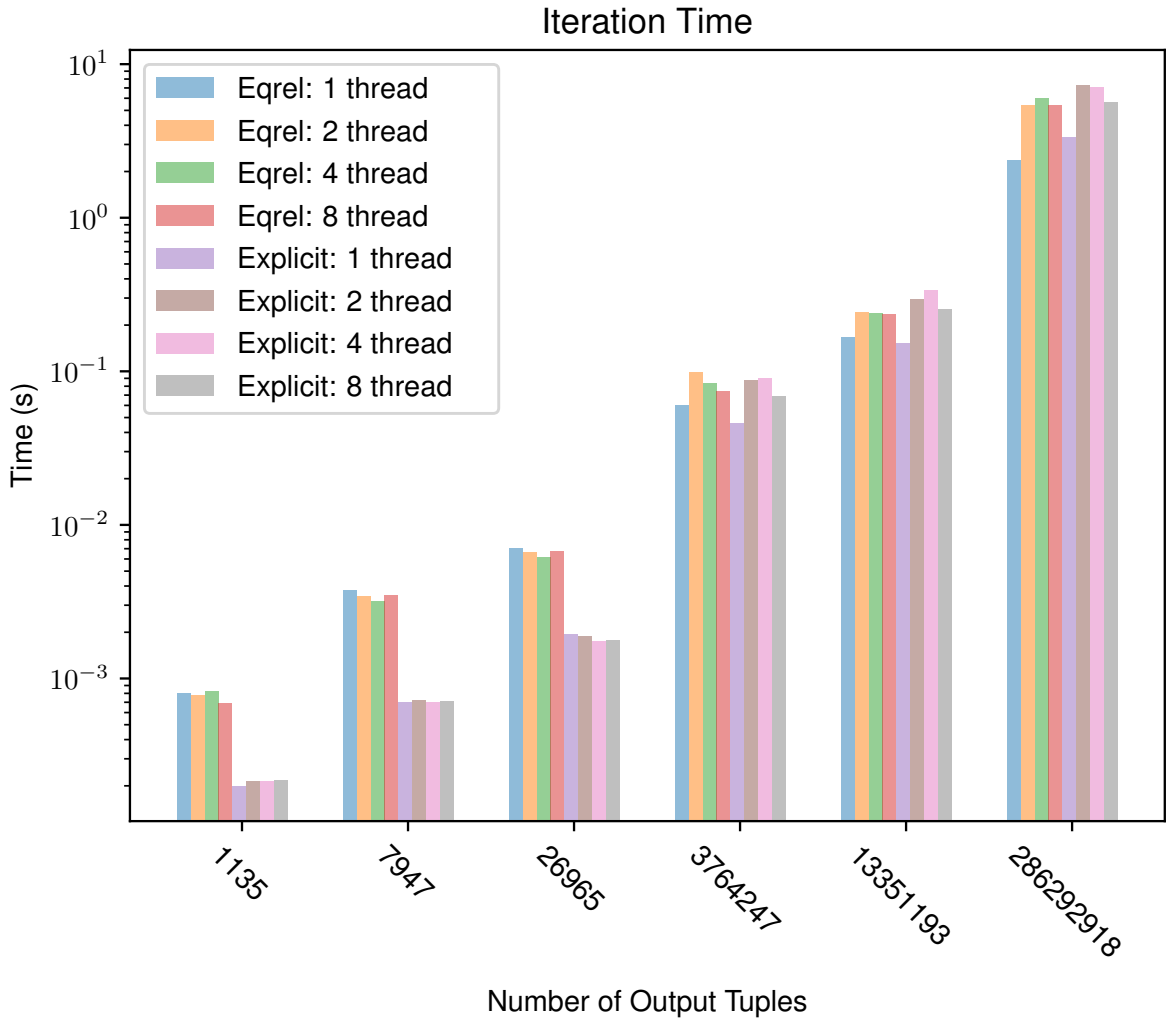


FIGURE 4.8: Iteration time versus the number of output tuples

It is interesting to note that the iteration time of the single-threaded implementation is consistently equal or better than the parallel iteration, despite it being intuitive that the threads are performing work independently. We are currently unsure of where this discrepancy lies, as it exists in both eqrel and explicit programs which may indicate a internal SOUFFLÉ issue. Internal profiling may be required to resolve the root cause. This time may also not be indicative to the actual iteration time over the pairs, as we increment an atomic integer to count the number of output tuples, as well as preventing the loop from being optimised out. In fact, this atomic integer may be the cause for the slowdown over multiple threads,

as despite our usage of the relaxed memory consistency model for this variable, the x86 architecture has strong memory guarantees as a baseline standard (Preshing, 2012) so contention over this variable may cause excessive synchronisation, and thus a slowdown in execution. If this is the case, it would also explain performance discrepancies with the performance of increasing parallelism with PiggyList.

For profiling the memory consumption, we measure the peak RSS (resident set size) memory usage, which incidentally also includes the I/O symbol table. Figure 4.9 demonstrates the increase in the memory consumption of programs, with Table 4.3 detailing the figures.

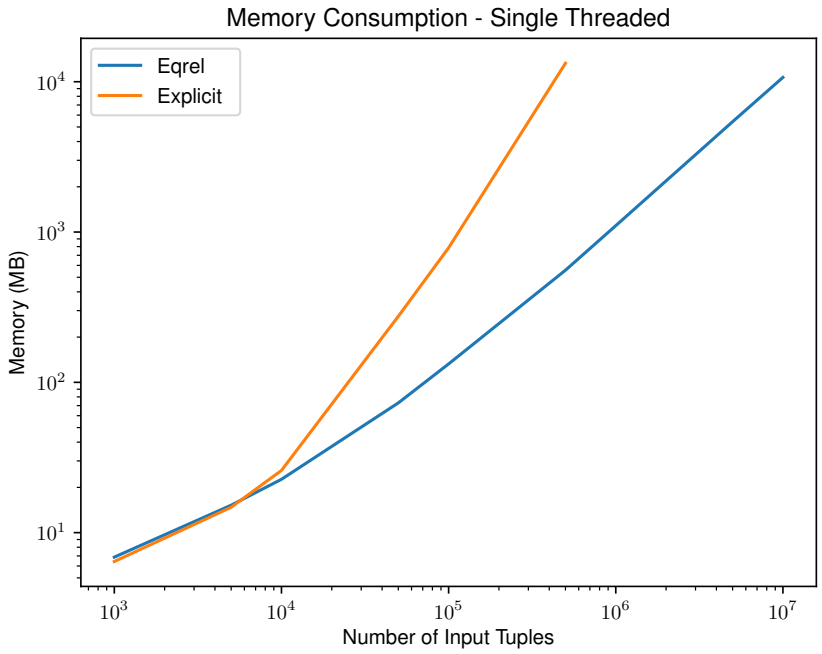


FIGURE 4.9: Memory use for both representations versus the number of input tuples

Input Size	Memory Consumption (MB)	
	Implicit	Explicit
1000	6.9	6.4
5000	15	15
10000	23	26
50000	73	270
100000	130	790
500000	560	13000
1000000	1100	-
5000000	5400	-
10000000	11000	-

TABLE 4.3: Memory Consumption of the Bitcoin Benchmarking Programs (MB, 2 s.f.)

Based on the figures, we observe a roughly linear increase in the memory consumption for the implicit representation, whereas for the explicit representation the growth appears roughly quadratic.

4.1.4 Discussion

We observe a near-quadratic runtime for the explicit representation, while the implicit representation was approximately linear, as was expected. This demonstrates that on real-world data-sets, even for those with medium-sized equivalence classes, quadratic run-times for the explicit representation will be observed, and thus using the implicit representation will dramatically improve solving performance for programs containing equivalence relations.

This trend was also observed for space consumption - we demonstrate that an explicit representation does in fact take quadratic space to represent, as was difficult to show in Section 3.1.5 as the time complexity of the explicit representation made benchmarks impractical.

A simple single-threaded Python imperative implementation took around 41 seconds to perform Union-Find over the 10 million input domain. The declarative implementation is only slower by a factor of 2, a narrower gap than usual - typical declarative programs are often magnitudes slower than their imperative equivalents. (Kastrinis et al., 2018) SOUFFLÉ was originally designed to provide similar performance to imperative implementations, and to be simpler to write. (Scholz et al., 2016)

4.2 Steensgaard Analysis

In order to statically analyse a program, the notion of memory modelling must be explored. One form of modelling is points-to analysis, which computes the objects that variables can point to over the duration of the program. As the amount of memory that will be allocated during a program could be unbounded (as is the case in Snippet 4.3), we require a memory abstraction. We treat all objects allocated at a program site to be interchangeable, and represent them by a single symbol. This method is known as representing each dynamic heap object their *abstract location*. (Sridharan et al., 2005) In Snippet 4.3, we may say the variable x points to o_4, where o_i refers to the object allocated on line i.

LISTING 4.3: Unknown quantity of memory allocation

```
1 int* foo() {
2     int* x;
```

```
3
4     while(condition) x = (int*) malloc(sizeof int);
5
6     return x;
7 }
```

This form of memory modelling was introduced by Andersen, (Andersen, 1994) where interactions between variables can be modelled via subset-constraints on their respective points-to sets. If a variable has the assignment `y = z`, then we say that the points-to set of `y` presumes all points-to elements of `z` - that is, $pts(y) \supseteq pts(z)$, where $pts(y)$ denotes the points-to set of `y`. This form of analysis was initially performed as flow-insensitive and context-insensitive meaning that the flow (or ordering of executed statements) of the program nor the execution contexts (i.e. where/what called the functions) are considered. This analysis carries a worst case cubic complexity (Shapiro and Horwitz, 1997); a less accurate, but significantly faster analysis was proposed by Steensgaard that ran in $\mathcal{O}(\alpha n)$ time, using union-find data-structures. (Steensgaard, 1996) This analysis carries the same flow- and context-insensitivity and Andersen's original analysis. The Steensgaard is much less accurate for a points-to analysis as it *merges* points-to sets on assignment, rather than retaining it as a subset constraint. This has the benefit of being trivially representable within a union-find data-structure (objects that share the same points-to set are within the same set within a disjoint-set structure), however will result in extremely large overheads.

Adding either flow- or context-sensitivity to a points-to analysis increases the usefulness of the points-to analysis, (Lhoták and Hendren, 2006) however the computational cost of such an analysis dramatically increases, although specialised methods exist to compute these. (Whaley and Lam, 2004)

4.2.1 Field-Sensitive Analysis

In addition to these, a *field-sensitive* analysis allows writes and reads to fields within records or objects to be disambiguated from other fields. Bodik & Sridharan proposed a simple flow-sensitive analysis, (Sridharan et al., 2005) that performs conditional points-to - if a variable `a` *loads* from a field `b1.f`, whilst a variable `c` stores to field `b2.f`, then `a` may point-to the same objects that `c` points to, if `b1` and `b2` point to shared memory. When two variables are said to point-to the same memory (i.e. the intersection of their points-to sets is non-empty), they are said to *alias*.

In Table 4.4, we construct the Bodik field-sensitive analysis, in addition to a modified Steensgaard analysis, which adds conditional assignment over aliasing fields.

Description	Assignment	Constraint	
		Bodik	Steensgaard*
Allocation	`a = new o()`	$o \in pts(a)$	$o \in pts(a)$
Assignment	`a = b`	$pts(a) \supseteq pts(b)$	$pts(a) = pts(b)$
Conditional Alias	`a = b1.f; b2.f = c`	$alias(b1, b2) \Rightarrow pts(a) \supseteq pts(c)$	$alias(b1, b2) \Rightarrow pts(a) = pts(c)$

TABLE 4.4: Constraint rules for the simplified field-sensitive language grammar

For the following program in Snippet 4.4, the corresponding to field-sensitive points-to set is depicted in Figure 4.10. For this example, both Bodik and Steensgaard* analyses result in the same points-to set. On line 5, the variable `z` is stored to `y.f`, whilst on line 6 the variable `w.f` is loaded from into `v`. As `y` and `w` alias (they both point to at least one shared object), the conditional alias holds, and thus we perform the equivalent operation for `v = z`. For our simplified Steensgaard analysis, we merge the two points-to sets of `z` and `v`, whilst for Bodik, we apply a subset constraint.

```
1 x = new o(); // o1
2 z = new o(); // o2
3 w = x;
4 y = x;
5 y.f = z;
6 v = w.f;
```

LISTING 4.4: Example field-sensitive program (Sridharan et al., 2005)

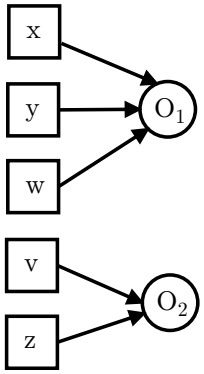


FIGURE 4.10: Resulting points-to set - field sensitive analysis

These Steensgaard analyses have typically had difficulty in matching the expected near-linear time when applied in a declarative context. Steensgaard relies on disjoint-sets which is by, implementing union-find, a destructive data-structure and thus is difficult to directly represent purely declaratively. Very recently, there have been promising results from expressing the union-find data-structure as purely declarative statements within the LogicBlox Engine. (Balatsouras et al., 2017) We wish to explore this in future

work.The equivalence relation that is induced by the use of the equality-based set operations can be simulated by the three explicit rules for reflexivity, symmetry, and transitivity, at the expense of efficiency.

However, for this input size it is not feasible to compute the Steensgaard analysis with the explicit representation. As the explicit representation run-time is proportional to the total number of output tuples stored (observed in Section 4.1), and the output tuples for the Steensgaard program is a trillion, a rough back of the envelope calculation shows it would take over 6 years to calculate (14 hours for the 280 million tuple program $\Rightarrow$ 14 hours $\times \frac{1151866748812}{286292918} \approx 6.6$ years).

4.2.2 Datalog Programs

We perform this benchmark using for two programs; a field-sensitive Steensgaard analysis written using the implicit equivalence relations (Snippet 4.6), and a field-sensitive Bodik analysis written with subset constraints (Snippet 4.7). All files share the same header, which declares the input rules (Snippet 4.5).

LISTING 4.5: Common Input Rules

```
1  // o: x := new T()
2  .decl alloc(x:symbol, o:symbol)
3  // x := y;
4  .decl assign(x:symbol, y:symbol)
5  // x := y.f;
6  .decl load(x:symbol, y:symbol, f:symbol)
7  // x.f := y;
8  .decl store(x:symbol, f:symbol, y:symbol)
9
10 // input EDB as CSV files
11 .input alloc, assign, load, store
```

Instead of iterating over all the derived tuples (as they vary massively in size between analyses), we simply print the size of the relation, which is the number of tuples stored. We mark the var points-to set (vpt) as eqrel to act an equivalence relation and thus represent tuples implicitly.

LISTING 4.6: Steensgaard Field-Sensitive Analysis in SOUFFLÉ

```
1  .decl vpt(x:symbol, y:symbol) eqrel
2  // allocation sites
```

```
3  vpt(x,y) :-
4    alloc(x,y).
5
6  // assignments
7  vpt(x,y) :-
8    assign(x,y).
9
10 // load/store pairs
11 vpt(y,p) :-
12   store(x,f, y),
13   load(p,q,f),
14   vpt(x,q).
15
16 // output computation
17 .printsize vpt
```

In the Bodik analysis, we can simply express subset constrains via Datalog rules. The assignment (`assign`) rule (line 9) dictates that for an assignment `x = y`, if an object is within `y`'s vpt set, it is now within `x`'s. We see a similar pattern for the load/store pair constraint (line 14), that for the object that is stored to (`x.f`) and loaded from (`q.f`) if they alias (they share an element within the vpt set), then we treat it as an assignment `p = y` and thus update the vpt set as per the assignment constraint.

LISTING 4.7: Bodik Field-Sensitive Analysis in SOUFFLÉ

```
1  // IDB
2  .decl vpt(x:Variable, o:Object)
3
4  //// allocation sites
5  vpt(x,o) :-
6    alloc(x,o).
7
8  // assignments
9  vpt(x,o) :-
10   assign(x,y),
11   vpt(y,o).
```

```
12
13 // load/store pairs
14 vpt(p,o2) :-
15   store(x,f, y), // x.f := y
16   load(p,q,f), // p := q.f
17   vpt(x,o1),  // alias x q
18   vpt(q,o1),
19   vpt(y,o2).
20
21 // output computation
22 .printsize vpt
```

4.2.3 Results

Loading in the facts from file consistently takes $\approx$ 5 seconds, and 992MB of memory. This consists of four fact files: `alloc.facts` (58MB), `assign.facts` (275MB), `load.facts` (31MB), and `store.facts` (9.8MB). This is the same for both Steensgaard and Bodik analyses. Running the two results in vastly different points-to sets - as Steensgaard is an over-approximation of the points-to set as compared to Bodik. The final count for Steensgaard is 1151866748812 (1.1 trillion) tuples, whilst for Bodik 389210 output tuples are generated. We believe this is the first Datalog program to have stored a trillion tuples, as Steensgaard analyses heavily favour imperative implementations and thus have not seen implementation in declarative languages before.

The solving runtime for the analyses is demonstrated in Figure 4.11 and Table 4.5.

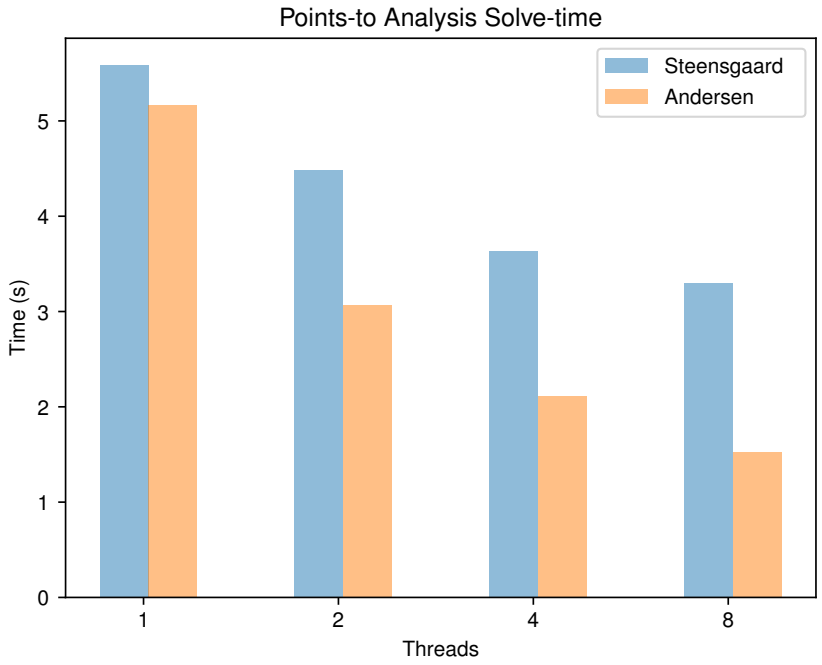


FIGURE 4.11: Solving time for various thread counts for each analysis

Threads	Analysis	
	Steensgaard	Bodik
1	5.6	5.2
2	4.5	3.1
4	3.6	2.1
8	3.3	1.5

TABLE 4.5: Solving time for the points-to analyses (seconds, 2 s.f.)

We can see that the Steensgaard analysis takes longer to solve than the Bodik analysis, and that the gains from adding additional threads does not significantly improve the running time as compared to the Bodik analysis. We can partition the solving program into several stages for each analysis, as is observed within the generated C++ code:

Steensgaard:

(1) `alloc` - insert all rules from the alloc relation

(2) `assign` - insert all rules from the assign relation

(3) `extend` - insert knowledge learned from the delta into the current knowledge

(4) `insert all` - used to generate a delta relation for the next step

(5) `load store` - perform the semi-naïve evaluation on this relation until fixed-point

From manual analysis of the generated C++ code, we see that the `extend` is unnecessarily inserted in this case, as no information has been loaded into the relation as used passed into the extend operation. Omitting this reduces the run-time, but may not necessarily be done automatically, as it depends on the program loaded in and thus may introduce incorrect behaviour if applied generally.

Bodik:

(1) `alloc` - insert all rules from the alloc relation

(2) `insert all` - used to generate a delta relation for the next step

(3) `mixed (assign & load store)` - perform the semi-naïve evaluation until fixed-point

As the `assign` and `load/store` rules are both recursive, these are performed in the same loop, until a fixed-point is reached, as new knowledge in one may generate knowledge in the other rule.

Due to this structural difference in the resulting program, we present the time breakdown for the above sections in Table 4.6.

Threads	Steensgaard					Bodik		
	`alloc`	`assign`	`extend`	`insert all`	`load store`	`alloc`	`insert all`	`mixed`
1	0.34	1.4	1.0	2.0	0.81	0.23	0.19	4.7
2	0.27	0.94	0.78	1.8	0.67	0.15	0.16	2.8
4	0.20	0.61	0.58	1.8	0.44	0.13	0.23	1.8
8	0.18	0.45	0.52	1.8	0.37	0.13	0.19	1.2

TABLE 4.6: Time breakdown of the points-to analysis

As mentioned, by removing the unnecessary `extend` section of the generated code, we are able to save up to a second. The portion of generated code that takes the longest time is the `insert all` snippet. This also demonstrates a poor threaded scaling - this is due to it not being implemented via OpenMP. It is possible to do so, but was omitted based on incorrect assumptions (as this demonstrates). It would be beneficial to repeat this benchmark again with this changed. These changes would bring the Steensgaard implementation to only have a marginal overhead over the Bodik analysis.

As demonstrated in Figure 4.12, the memory overhead for the Steensgaard analysis is comparable to the computational overhead.

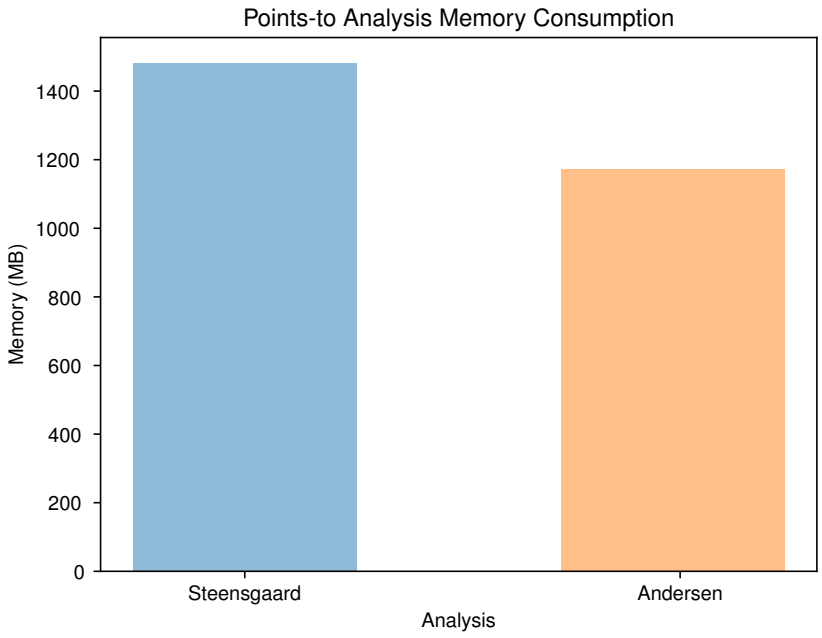


FIGURE 4.12: Maximum resident memory size for each analysis

We believe the potential changes made above would not have an effect in the overall memory consumption.

4.2.4 Discussion

Steensgaard analysis was proposed to combat the associated cost of Bodik analysis, (Steensgaard, 1996) due to the poor computational power at the time of their introduction in the 1990s. Since then computational power has dramatically improved, and as such, the runtime of an Bodik analysis (worst-case cubic) is negligible on modern hardware.

However, despite the recent rise of declarative languages for performing of points-to analysis, Steensgaard analyses were still strictly confined to imperative implementations due to their nature. (Kastrinis et al., 2018) In demonstrating similar performance we believe we have proven that Steensgaard analyses are now reasonable within Datalog.

There are multiple optimisations that could be made, based on the above performance analysis, with the performance breakdown indicating that the lack of parallelism for some operations significantly contribute to the running time for larger programs.

CHAPTER 5

Future Work

Based on both the internal micro-benchmarks (i.e. benchmarking the individual layers) and the full system benchmarks showed areas of improvement. These may potentially be both in the algorithms for each operation, or, within the choice of data-structures and potential micro-optimisations. As the program is highly repetitive in that some functions are called millions, billions, or even trillions of times, minor improvements in code may correspond to significant improvements in memory usage or runtimes.

As noted in the related work, a recent paper *An Efficient Data Structure for Must-Alias Analysis* by Kastrinis et al. investigates the use of a declarative union-find program for a points-to analysis - that is, union-find was implemented via pure Datalog statements. Writing equivalent SOUFFLÉ programs and comparing the runtime to our implicit implementation is a highly relevant area of further research. We were not able to perform this due to the implicated time constraints with such a recently published paper.

We note further areas of research and improvement that could be made in the various layers of the data-structure.

5.1 Equivalence Relation

Currently, users of SOUFFLÉ must must tag their relations as `eqrel` in the declaration in order to generate the implicit behaviour. It is possible to perform an analysis on the relations to detect equivalence relations automatically via detecting reflexive, symmetric, and transitive rules. Pattern matching is one way to achieve this, however this may not capture all equivalence relations. This behaviour may be unwanted, as some SOUFFLÉ features were not designed for use with implicit relations as this implementation is the first example of such functionality. This interfered with separate features such as provenance (i.e. computing *which* rules were used to compute a fact), and magic-set optimisations - discussion with the

other developers of SOUFFLÉ indicates this is primarily due to problems in the implementation of, and not the theoretical limitations of, these features.

Some functions proved to impart a substantial performance impact, or, not a great improvement in performance as the parallelism factor was increased. As explored in the Steensgaard points-to benchmark, we observe several functions or portions of the generated code that interact with the equivalence relation do not enjoy as linear an improvement as the comparative Bodik analysis. Investigating this with additional internal profiling may reveal the root cause.

Due to time constraints, we were not able to investigate the cost of generating the equivalence cache, or whether threading the internals of this function would yield considerable benefits. We also wish to investigate additional kinds of iterators for the partitioning - namely a multi-set closure, when dealing will small equivalence classes. There were also stability issues for the extremely large Bitcoin datasets, with almost 20% of runs crashing due to segfaults or exceptions.

Based on the improved performance demonstrated in the benchmarks of the Libcuckoo map over the Intel TBB map, it would be worth investigating the potential run-time improvements in also integrating this map.

The investigation of the perceived overhead of the over-approximation of the delta knowledge has not been detailed. Depending on this, it may result in the ability of other fast, approximation strategies for other data-structures for relations in SOUFFLÉ .

5.2 Sparse Mapping

In internal profiling we observed the sparse mapping layer to take a significant portion of execution, namely the hash-map queries and operations introduced significant constants into the runtime. These hash-maps are highly optimised for general use, however for improved performance we may benefit from stripping functionality and implementing custom logic. As a key may only be set once, we may be able to optimise the internal hash-map logic.

We were not able to explore lock-stratification as an contention strategy for some of the hash-maps. By doing so, we may see improved performance above the optimistic dense element allocation strategy.

Due to unresolved bugs, we were not able to complete the integration of the Libcuckoo hash-map into our Densifier layer. Based on our micro-benchmarking we saw this hash-map performed better than the Intel hash-map by 95% in some cases. Integrating this, and re-running the benchmarks may result in major improvements, potentially enough to make the Steensgaard analysis perform faster than the Bodik analysis.

In general, this layer did not see a drastic improvement as the parallelism factor increased. In fact, the single-threaded performance was often better than all multi-threaded runs. Online benchmarks of the data-structures tended to show a linear, if not super-linear improvement. (Preshing, 2016c) Profiling may reveal the underlying cause, and consequently offer a performance boost.

5.3 PiggyList

Further investigation and formalisation of the wait-free versions and reduction in memory 'bottlenecks' should be performed. Furthermore, within the literature on wait-free data-structures, there appears to little discussion on space complexities. The probabilistic approach yield favourable results in application to other existing wait-free data-structures.

As the method has proved viable for PiggyList with a simple linear dice-roll function, exploring additional functions may result in a much-lower average overhead. At an extreme, we may be able to procedurally generate empirically 'optimal' (i.e. no recorded overhead) functions through reinforcement learning.

For all versions, we observed that the data-structure did not see a super-linear speed improvement. We suspect this is due to the atomic synchronisation that is implicit on x86 architectures, even when specifying relaxed memory models. Running the PiggyList benchmarks on a CPU architecture with a weaker memory model, such as ARM, may confirm this.

Conclusion

In this work we designed and implemented concurrent data-structure for logical equivalence relations. To deploy a self-computing data-structure that performs rules for reflexivity, symmetry, and transitivity implicitly, the semi-naïve evaluation strategy had to be changed. In addition, we designed a layered data-structure architecture to provide a seamless integration of an equivalence relation in a Datalog engine such as SOUFFLÉ . The data-structure was designed for parallel execution so that large volumes of data can be processed efficiently.

The data-structure has three layers. The first layer provides a relational interface to the Datalog engine. This layer provides fast iterations over the implicitly stored equivalence pairs. The key insight for performance for the first layer was the construction of a cache to support parallel iteration (whilst guaranteeing no overlapping domains) in a performant manner.

The second layer condenses the domain of the logical relation to an ordinal domain. The ordinal domain provides an efficient storage of elements in the subsequent find-union data-structure. The implementation of the second layer is a bijective map between these sparse and dense values.

The third layer of the data-structure is a parallelised union-find data-structure. This data-structure grows monotonically and we were required to implement a parallel array called PiggyList to utilise the parallel growth of data. The PiggyList has been shown to extend to wait-free guarantees, showing that the wait-free guarantee of find-union can be retained even for non-fixed domains.

We have demonstrated the efficacy of our implementation in SOUFFLÉ through application to real-world benchmarks. For a Bitcoin data-set, we compared the performance of the implicit equivalence relation representation to an explicit representation which confirmed a quadratic speed-up in computing time, and equivalent reductions in memory. For a points-to analysis, we show that a Steensgaard analysis can be performed with approximately equivalent performance to an inclusion based analysis. The Steensgaard analysis completed in under 4 seconds, and generated a trillion tuples. We estimated that computing the explicit representation on the same dataset would take several years.

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